The Continuum Hypothesis [CH] is neither a definite mathematical problem nor a definite logical problem

Solomon Feferman, Logic Seminar, Tues. 11/17/15

**Source:** Revised article for the EFI project, <http://math.stanford.edu/~feferman/papers/CH_is_Indefinite.pdf>.

**1. The two faces of the continuum problem.**

What is the nature of mathematical problems? What is the nature of logical problems?

My main claim re CH.

**2. The metamorphosis of CH from a mathematical problem to a logical problem.**

The evidence for the transformation of CH from a mathematical problem to a logical problem.

What mathematicians in general know and don’t know about the work on CH.

**3. Background: The road to large cardinals; their logical template.**

Scott’s theorem and Cohen’s results.

“Small” large cardinals and “large” large cardinals.

The Lévy and Solovay theorem.

The template for “large” large cardinals

Large Cardinal Axioms (LCAs)

>[Insert Steel, “What are Woodin cardinals?”]

**4. Why accept LCAs? I. The consistency (or interpretability) hierarchy.**

Kunen’s theorem.

The consistency and interpretability hierarchies.

The observed central role for LCAs.

Woodin’s view (more or less): all LCAs are true.

**5. Why accept LCA’s? Mathematical consequences**

Descriptive set theory (DST); the “regularity” properties, PSP.

The intermediate role of the Axiom of Determinacy (AD); but AD implies not-AC.

Weakenings of AD: AD, ADDef, PD, ADL(R),

The Martin and Steel theorem; Woodin’s theorem.

LCAs and ADDefs

Who’s on board with this?

**6. Is CH a definite logical problem?**

The -logic Programand the Inner Model Program.

-logic; semantic and “syntactic” notions of consequence.

The -Conjecture, the Strong -Conjecture, and the CH conjecture.

Inner models and the aim of the Inner Model Program

L[E], L[E]S, L[E](\*)

CH is not a definite problem for these programs.

>[Other programs?]

**7. The “duck” problem**

What is the duck problem?

Why it’s not a problem for Gödel.

Why it’s a problem for current proponents of higher set theory.

My own views; conceptual structuralism, definite and indefinite problems.

>[Subsequent discussion]