I/O Logic

Leendert (Leon) van der Torre

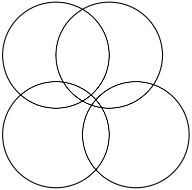
Joint work with David Makinson and Xavier Parent

Layout of this talk

Input/output logic

1. Motivation: DSDL family
2. Motivation: Makinson@DEON98
3. Unconstrained
4. Permission
5. Constrained
6. Challenges

“Exotic Logics”



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- D. Lewis, *Counterfactuals*, 1973, Blackwells.
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- D. Makinson, “Five faces of minimality”, *Studia Logica*, 52, 1993, pp. 339-379.
- D. Makinson, Bridges from Classical to Nonmonotonic Logic. College Publications, 2005.
- D. Gabbay et al, Handbook of deontic logic, 2013.

Related to the area of preference logic

Language DSDL family

$$A ::= p \mid \neg A \mid A \wedge B \mid \Box A \mid \bigcirc(B/A) \mid P(B/A)$$

New building blocks

- $\bigcirc(B/A) = B$ is obligatory, given A
- $P(B/A) = B$ is permitted, given A

Language DSDL family

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- $P(B/A) = B$ is permitted, given A

May also be written as:

- Counterfactual logics $A \Rightarrow B$, or
- Non-monotonic logic $A \mid\!\sim B$

Relation (some) preference logics:

- $A \geq B = \neg O(\neg A | A \vee B)$ and $O(A|B) = (A \wedge B) > (\neg A \wedge B)$

- S. Kaci, Working with Preferences: Less Is More (Cognitive Technologies), Springer 2011.
- F. Rossi, K. Venable, T. Walsh, A Short Introduction to Preferences: Between AI and Social Choice. Morgan Claypool, 2011.

Model DSDL family

Model

$M = (W, \geq, V)$, with

- W: a set of possible worlds $\{x, y, ...\}$
- $\geq$: binary relation ranking all possible worlds in betterness
  - $x \geq y$: x is al least as good as y
- V is valuation function as usual

Note: the ranking can be made world-relative too.

Evaluation rules DSDL family

Evaluation rules

$M, x \models \bigcirc(B/A)$ iff $\mathrm{opt}_{\succeq}(\|A\|) \subseteq \|B\|$
$M, x \models P(B/A)$ iff $\mathrm{opt}_{\succeq}(\|A\|) \cap \|B\| \neq \emptyset$
where

- $\|A\| = \{x \in M : x \models A\}$
- $\mathrm{opt}_{\succeq}(X) = \{x \in X : (\forall y \in X) x \geq y\}$

P dual of $\bigcirc$, i.e., $P(B/A) = \neg \bigcirc (\neg B/A)$

Classes of structures DSDL

Constraints on $\geq$

Reflexivity: $x \geq x$

Transitivity: $x \geq y$ and $y \geq z$ implies $x \geq z$

Totalness: $x \geq y$ or $y \geq x$

Limit assumption: no infinite sequence of strictly better worlds

$$\|A\| \neq \emptyset \rightarrow \mathrm{opt}_{\succeq}(\|A\|) \neq \emptyset$$

	constraints on $\geq$
DSDL1	reflexivity
DSDL2	reflexivity, and limit assumption
DSDL3	reflexivity, transitivity, totalness, and limit assumption

Table 1: Hansson 1969's systems

Full axiomatization Total order case

Åqvist's System G

All truth functional tautologies	(PL)
S5-schemata for $\Box$ and $\Diamond$	(S5)
$P(B/A) \leftrightarrow \neg \bigcirc (\neg B/A)$	(DfP)
$\bigcirc (B \to C/A) \to (\bigcirc(B/A) \to \bigcirc(C/A))$	(COK)
$\bigcirc (B/A) \to \Box \bigcirc (B/A)$	(Abs)
$\Box A \to \bigcirc(A/B)$	(CON)
$\Box(A \leftrightarrow B) \to (\bigcirc(C/A) \leftrightarrow \bigcirc(C/B))$	(Ext)
$\bigcirc (A/A)$	(Id)
$\bigcirc (C/A \wedge B) \to \bigcirc(B \to C/A)$	(C)
$\Diamond A \to (\bigcirc(B/A) \to P(B/A))$	(D*)
$(P(B/A) \wedge \bigcirc(B \to C/A)) \to \bigcirc(C/A \wedge B)$	(S)
If $\vdash A$ and $\vdash A \to B$ then $\vdash B$	(MP)
If $\vdash A$ then $\vdash \Box A$	(N)

Selected bibliography (1)

- L. Aqvist, *Introduction to Deontic Logic and the Theory of Normative Systems*, Napoli, Bibliopolis, 1987.
- L. Aqvist, "Deontic Logic". In D. Gabbay and F. Guenthner (Eds.), *Handbook of Philosophical Logic*, 2nd Edition, Vol. 8, pp. 147-264, Kluwer Academic, 2002.
- M. Ardeshir and F. Nabavi, On some questions of L. Åqvist, *Logic Jnl IGPL*, 14, 2006, pp. 1-13.
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- J. Hansen, "On relations between Aqvist's deontic system G and Van Eck's deontic temporal logic". In P. McNamara and H. Prakken (Eds.), *New Studies in Deontic Logic and Computer Science*, Vol. 49 of *Artificial Intelligence and Applications series*, IOS Press, Amsterdam 1998, pp. 127-144.

Selected bibliography (2)

- B. Hansson, "An analysis of some deontic logics", *Nous*, 3, 1969, pp. 373-398.
- D. Lewis, *Counterfactuals*, 1973, Blackwells.
- D. Makinson, "Five faces of minimality", *Studia Logica*, 52, 1993, pp. 339-379.
- X. Parent, "On the strong completeness of Aqvist's dyadic deontic logic G". In van der Meyden and van der Torre (eds), *Deontic Logic in Computer Science 9th International Conference*, DEON 2008, Luxembourg, Luxembourg, July 15-18, 2008. Proceedings, pp. 189-202.
- X. Parent, "A complete axiom set for Hansson's system DSDL2", *Logic Jnl IGPL*, 18(3), 2010, pp. 422-429.
- W. Spohn, "An analysis of Hansson's dyadic deontic logic", *Journal of Philosophical Logic*, 4, 1975, pp. 237-252.

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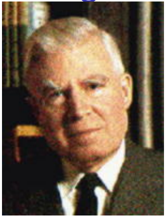
Von Wright@DEON98

Deontic logic—as I see it

Does not even mention preferences:

- "[The philosophical aspects] have made me feel more and more like a lonely wolf . . . "
- "Logic has a wider reach than truth."
- "The only acceptable answer I can think of must make reference to the purpose or rationale of norm giving activity."
- "A consistent set of norms entails a further norm if, and only if, adding to the set the negation norm of this further norm makes the set inconsistent."
- "A problem child in the philosophy of norms has been the notion of permission."

Makinson@DEON98

A fundamental problem of deontic logic

- Jorgensens dilemma (1931)
  "A fundamental problem of deontic logic, we believe, is to reconstruct it in accord with the philosophical position that norms direct rather than describe, and are neither true nor false."
  "No logic of norms without attention to a system of which they form part." (iterative approach)
- "Validation of formulae $O(b/b)$ whose intuitive standing is open to question."
- "It is rather difficult to construct an account of what is known as strong or positive permission along the same lines"
- Mobius strip (from nonmonotonic logic) $O(\neg a|c)$, $O(c|b)$, $O(b|a)$ in context a

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Language

- A conditional obligation is a pair (a, x), called a generator
- a and x are propositional formulae
- (a, x) is a rule; a is called the body and x is called the head
- A normative system N is a set of such pairs

Methodology: Develop examples and let the theory emerge

Semantics based on detachment

The semantics is 'operational'

$$\boxed{x \in out(N, A)}$$

- Calculates whether according to normative system N and in context A, a formula x is obligatory
- A is the input (a set of wffs); x is the output
- Detachment as a core mechanism

Modus-ponens

(i)	If a, then x
(ii)	a
(iii)	So, x

Boghossian:
this is constitutive of the notion of conditional

Detachment
- the only assumption made in IOL
- can hardly be challenged

DSDL
- extra assumptions
- potentially prone to criticisms
  - maximizing
  - trichotomy of value relations

Detachment

(1) Factual detachment

(i) If a is the case, then x is obligatory
(ii) a is the case
(iii) So, x is obligatory

In the I/O notation:

$$\text{If } (a, x) \in N \text{ then } x \in out(N, a)$$

N: a set of pairs of propositional wffs (conditional norms)
$out(N, a)$: output of a under N.

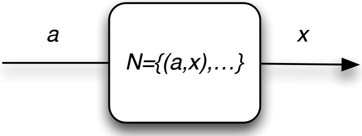


Figure 1: Factual detachment

Simple-minded output operation out_1

Simple-minded output, out_1

$$G(X) =_{df} \{x : (a, x) \in N \text{ for some } a \in X\}$$

$$out_1(N, A) =_{df} Cn(G(Cn(A))$$

Simple-minded output operation out_1

Simple-minded output, out_1

$$G(X) =_{df} \{x : (a, x) \in N \text{ for some } a \in X\}$$

$$out_1(N, A) =_{df} Cn(G(Cn(A))$$

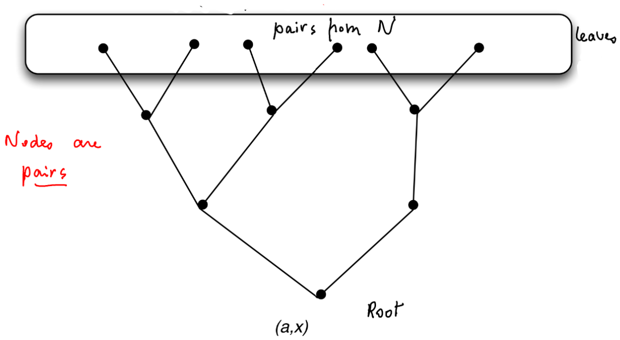
N	A	$Cn(A)$	$G(Cn(A))$	$Cn(G(Cn(A))$
$\{(a, x)\}$	a	$a, \ldots$	x	$Cn(x)$
$\{(a \vee b, x)\}$	a	$a \vee b, \ldots$	x	$Cn(x)$
$\{(a, x), (a, y)\}$	a	$a, \ldots$	x, y	$Cn(x, y)$

Proof-theory

Trick: $(a, x) \in out(N)$ in place of $x \in out(N, a)$.

Derivation

(a, x) derivable from N - notation: $(a, x) \in deriv_1(N)$ – iff (a, x) is in the least superset of N that includes $(\top, \top)$ and is closed under the rules of the system.



Rules

$$\text{(SI)} \quad \frac{(a,x) \qquad b \vdash a}{(b,x)} \qquad \text{(WO)} \quad \frac{(a,x) \qquad x \vdash y}{(a,y)}$$

$$\text{(AND)} \quad \frac{(a,x) \qquad (a,y)}{(a, x \wedge y)}$$

Table 2: IOL system

Output operation	Rules
Simple-minded (out_1)	{SI, WO, AND}

Completeness theorem

$x \in out_1(N, a)$ iff (a, x) derivable from N using these rules

Unpacking out_1

$x \in Cn(G(Cn(A))$ means

$A \vdash a_1 \wedge ... \wedge a_n$

$(a_1, x_1), ..., (a_n, x_n)$ in N

and $x_1 \wedge ... \wedge x_n \vdash x$

Some other I/O operations

- out_2: basic output
- out_3: reusable simple-minded
- out_4: reusable basic

$$\text{(SI)}\quad \frac{(a,x) \qquad b \vdash a}{(b,x)} \qquad\qquad \text{(WO)}\quad \frac{(a,x) \qquad x \vdash y}{(a,y)}$$

$$\text{(AND)}\quad \frac{(a,x) \qquad (a,y)}{(a, x \wedge y)} \qquad\qquad \text{(OR)}\quad \frac{(a,x) \qquad (b,x)}{(a \vee b, x)}$$

$$\text{(CT)}\quad \frac{(a,x) \qquad (a \wedge x, y)}{(a,y)}$$

Table 3: IOL systems

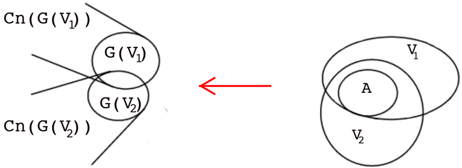
Output operation	Rules
Simple-minded (out_1)	{SI, WO, AND}
Basic (out_2)	{SI, WO, AND}+{OR}
Reusable simple-minded (out_3)	{SI, WO, AND}+{CT}
Reusable basic (out_4)	{SI, WO, AND}+{OR,CT}

Basic output, out_2

Call a set V of wffs complete iff: $V = L$ (the set of all wffs) or V maximal consistent.

Basic output

$out_2(N, A) = \cap\{Cn(G(V)) : A \subseteq V, V \text{ complete}\}$



V	$G(V)$	$Cn(G(V))$
V_1	$G(V_1)$	$Cn(G(V_1))$
V_2	$G(V_2)$	$Cn(G(V_2))$
$\vdots$	$\vdots$	$\vdots$
V_i	$G(V_i)$	$Cn(G(V_i))$

One of these V_i is L.
The others are maximal consistent extension (MCE) of A

Reasoning by cases

$$out_2(N, A) = \cap\{Cn(G(V)) : A \subseteq V, V \text{ complete}\}$$

$\boxed{N = \{(a, x), (b, x)\}.\ A = \{a \vee b\}}$

We have $out_2(N, a \vee b) = Cn(x)$.

V	$G(V)$	$Cn(G(V))$
L	$\{\mathrm{x}\}$	$Cn(x)$
MCE of $\{a \vee b\}$	$\{\mathrm{x}\}$	$Cn(x)$

Disjunction property: $a \vee b$ in a MCE iff one of the disjuncts in it.

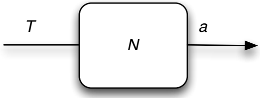
Iteration of successive detachments

(2) Deontic detachment

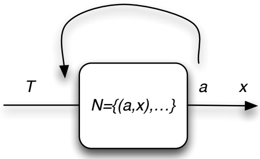
(i) If a is obligatory, then x is obligatory
(ii) a is obligatory
(iii) So, x is obligatory

In the I/O notation:

If $a \in out(N, \top)$ and $(a, x) \in N$ then $x \in out(N, \top)$



(a) a detached



(b) x detached

Figure 2: Iteration of successive detachments

Reusable output, out_3 and out_4

Reusable simple-minded output

$$out_3(N, A) = \cap\{Cn(G(B)) : A \subseteq B = Cn(B) \supseteq G(B)\}$$

Reusable basic output

$$out_4(N, A) = \cap\{Cn(G(V)) : A \subseteq V \supseteq G(V), V \text{ complete}\}$$

Example

- Show that GC is a rule of $deriv_4$

$$\frac{(a, b) \qquad (\neg a, c)}{(\neg c, b)}$$

Example

- Show that GC is a rule of *deriv*$_4$

$$\frac{(a, b) \qquad\qquad (\neg a, c)}{(\neg c, b)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{(\neg a,c)}{(\neg c\wedge\neg a,c)}\,\text{SI}\quad \dfrac{(a,b)}{(\neg c\wedge\neg a\wedge c,b)}\,\text{SI}}{(\neg c\wedge\neg a,b)}\,\text{CT}\qquad \dfrac{(a,b)}{(\neg c\wedge a,b)}\,\text{SI}}{((\neg c\wedge\neg a)\vee(\neg c\wedge a),b)}\,\text{OR}}{(\neg c,b)}\,\text{SI}$$

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Permission

- It is permitted to drive at a speed of 95 km/h on a motorway
- Anyone over 18 can buy booze legally

Permission

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Negative permission

Let N be a set of generators, and let *out* be an input/output logic.

$$(a, x) \in negperm(N) \text{ iff } (a, \neg x) \notin out(N)$$

Positive permission - static

Let N and P be two sets of generators, where P stands for permissive norms, and let *out* be an input/output logic.

$$(a, x) \in statperm(P, N) \text{ iff } (a, x) \in out(N \cup Q)$$
$$\text{for some singleton or empty } Q \subseteq P$$

(a, x) is generated either by the obligations in N alone, or by the norms in G together with some explicit permission (b, y) in P.

Proof theory

Subverse of a rule: obtained by downgrading to permission status one of the premises, and also the conclusion of the rule.
Ex: subverse of AND is

$$\frac{(a,x)^o \quad (a,y)^p}{(a,x \wedge y)^p}$$

Observation

If *out* satisfies a rule R, then the corresponding *statperm* satisfies its subverse.

Theorem

For each of the following rule-sets, the subverse set suffices to characterize the corresponding static permission operation.

1. out_1 with its usual rules SI, WO, AND,
2. out_2 with its usual rules, i.e. the above plus OR,
3. out_3 with the rules SI, WO, CTA:

$$\frac{(a,x),(a \wedge x, y)}{(a, x \wedge y)}$$

Permission

Positive permission - dynamic

$$(a, x) \in dynperm(P, N) \text{ iff } (c, \neg z) \in out(N \cup \{(a, \neg x)\})$$
$$\text{for some } (c, z) \in statperm(P, N) \text{ with}$$
$$c \text{ consistent}$$

Forbidding x under condition a would prevent the agent from making use of some explicit (static) permission (c, z).
(a, x) protected by the code.

Permission

Positive permission - dynamic

Danish's caricatures of the prophet Mohammed

freedom of speech

banning

$$(a, x) \in dynperm(P, N) \text{ iff } (c, \neg z) \in out(N \cup \{(a, \neg x)\})$$

$$\text{for some } (c, z) \in statperm(P, N) \text{ with}$$

$$c \text{ consistent}$$

publishing caricatures

freedom speech

Forbidding x under condition a would prevent the agent from making use of some explicit (static) permission (c, z).
(a, x) protected by the code.

Proof theory

Inverse of a rule: obtained by downgrading to permission status one of the premises, and also the conclusion of the rule, swapped them, and negating both their heads.
Ex: inverse of AND is

$$\frac{(a, x)^o \quad (a, \neg(x \wedge y))^p}{(a, \neg y)^p}$$

Observation

If *out* satisfies a rule R, then the corresponding *negperm* and *dynperm* satisfy its subverse.

Theorem

For each of the following rule-sets, the inverse set suffices to characterize the corresponding dynamic permission operation.

1. out_1 with its usual rules SI, WO, AND,
2. out_2 with its usual rules, i.e. the above plus OR,
3. out_3 with the rules SI, WO, CTA.

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Constrained I/O Logic (cIOL)

Problems

- uIOL not conflict-tolerant

  $N_1 = \{(a, b), (a, \neg b)\}$ input: a - output: L Explosion!!

Constrained I/O Logic (cIOL)

Problems

- uIOL not conflict-tolerant

  $N_1 = \{(a, b), (a, \neg b)\}$ input: a - output: L Explosion!!
- uIOL not violation-tolerant

  $N_2 = \{(\top, \neg a), (\neg a, \neg b), (a, b)\}$ input: a - output: L Explosion!!

Constrained I/O Logic (cIOL)

Problems

- uIOL not conflict-tolerant

  $N_1 = \{(a, b), (a, \neg b)\}$ input: a - output: L Explosion!!

- uIOL not violation-tolerant

  $N_2 = \{(\top, \neg a), (\neg a, \neg b), (a, b)\}$ input: a - output: L Explosion!!

Threshold idea

- Cut back the set of generators in N to just below the threshold of yielding excess

Norm violation

- C: a set of additional formulae called ‘constraints’. The output must be consistent with it.
- For CTDs, $C = A$.

Maxfamily

- $maxfamily(N, A, C)$ is the set of $\subseteq$-maximal subsets N' of N such that $out(N', A)$ is consistent with C.
- $outfamily(N, A, C) = \{out(N', A) \mid N' \in maxfamily(N, A, C)\}$.
- $x \in out_{\cup/\cap}(N, A)$ iff $x \in \cup/\cap\, outfamily(N, A, C)$

Chisholm example

h: help; t: tell

$$N = \{\underbrace{(\top, h)}_{❶}, \underbrace{(h, t)}_{❷}, \underbrace{(\neg h, \neg t)}_{❸}\}$$

$A = \{\neg h\}$

Chisholm example

h: help; t: tell

$$N = \{\underbrace{(\top, h)}_{❶}, \underbrace{(h, t)}_{❷}, \underbrace{(\neg h, \neg t)}_{❸}\}$$

$A = \{\neg h\}$

$maxfamily(N, A, A) = \{\{❷, ❸\}\}$ *and*
$out_{\cup/\cap}(N, A) = Cn(\neg t)$

Chisholm example

h: help; t: tell

$$N = \{\underbrace{(\top, h)}_{❶}, \underbrace{(h, t)}_{❷}, \underbrace{(\neg h, \neg t)}_{❸}\}$$

$A = \{\neg h\}$

$maxfamily(N, A, A) = \{\{❷, ❸\}\}$ *and*
$out_{\cup/\cap}(N, A) = Cn(\neg t)$

$A = \{h\}$

$maxfamily(N, A, A) = \{\{❶, ❷, ❸\}\}$ *and*
$out_{\cup/\cap}(N, A) = Cn(h, t)$

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Challenges

1. Drowning problem
2. Priorities and other extensions
3. Norm change
4. Dichotomy unconstrained/constrained and strong equivalence
5. Framework and requirements

Requirements

FD	$(\alpha,\beta) \in N \Rightarrow \beta \in out(N,\alpha)$	Factual Detachment
VD	$(A,\beta) \Rightarrow (A \cup \{\neg\beta\},\beta)$	Violation Detection
SUB	$\alpha \in out(N,A) \Rightarrow \alpha[\sigma] \in out(N[\sigma],A[\sigma])$	Substitution
RLE	$N \approx M, Cn(A) = Cn(B), Cn(\alpha) = Cn(\beta),$ $(A,\alpha) \in out(N) \Rightarrow (B,\beta) \in out(M)$	Replacement of equivalents
IMP	$out(N,A) \subseteq Cn(m(N) \cup A)$	Implication
PC	$\alpha \in \overline{V}(N,A) \Rightarrow \exists M \subseteq N : \alpha \in out(M,A)$ and $out(M,A) \cup A$ consistent	Paraconsistency
AND	$(A,\alpha)(A,\beta) \Rightarrow (A,\alpha \wedge \beta)$	Conjunction
FM	$(A,\alpha) \Rightarrow (A \cup B,\alpha)$	Factual monotony
NM	$out(N) \subseteq out(N \cup M)$	Norm monotony
NI	$M \subseteq O(N) \Rightarrow O(N) = O(N \cup M)$	Norm Induction

Deonticlogic.org

Gabbay et al, Handbook DL volume 1, 2013