Saussurean Compositionality

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Direct compositionality and triple grammars

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> *. . . the semantics works in tandem with the syntax: each syntactic rule which predicts the existence of some well-formed expression (as output) is paired with a semantic rule which gives the meaning of the output expression in terms of the meaning(s) of the input expressions. This is what we mean by Direct Compositionality.*
>
> *Thus every expression of a language—including the basic expressions (the words)—can be seen as a triple ⟨[sound], syntactic category, meaning⟩. ... A rule is thus something which takes one or more triples as input and yields one as output.* (Pauline Jacobson, *Compositional Semantics. An Introduction to the Syntax-Semantics Interface*)

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Hence this paper.

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Interestingly, Lakoff takes this to support non-compositionality.

> – *Syntactic categories are not autonomous, nor are they completely predictable form semantic considerations.*
> – *The meanings of whole grammatical constructions are not computable by general rules from the meanings of their parts.* (582)

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I will start with a few brief remarks on the second (larger and less precise) question.

Then I will address the first question in some detail, and draw a few conclusions.

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So grammar rules (or constraints) generate the well-formed signs, which constitute the language L.

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NB There are no corresponding terms on the meaning side.

Ambiguity in the standard account

By the standard account we mean syntax in the form of a grammar

$$\mathbf{E} = (E, A, f^{\mathbf{E}})_{f \in \Sigma}$$

which generates E from the atoms (lexical items) in $A \subseteq E$ via the functions (grammar rules) $f^{\mathbf{E}}$ (where Σ is a signature),

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A sign-based account, on the other hand, simply has the two distinct signs $\langle bank, m_1 \rangle$ and $\langle bank, m_2 \rangle$ in the language.

What about structural ambiguity?

On the standard account, terms (analysis trees) are used for structural ambiguity: e.g. with the rules

$$\begin{array}{ll} \mathsf{N} \rightarrow \mathsf{N}\ \mathsf{and}\ \mathsf{N} & (\text{rule } f) \\ \mathsf{N} \rightarrow \mathsf{A}\ \mathsf{N} & (\text{rule } g) \end{array}$$

and atoms *old*, *men*, *women*, we get two distinct terms:

$$\begin{aligned} t &= f(g(old, men), women) \\ u &= g(old, f(men, women)) \end{aligned}$$

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A compositional semantics μ yields

$$\mu(t) = r_f(r_g(m_o, m_m), m_w) = (m_o \cap m_m) \cup m_w = m$$
$$\mu(u) = r_g(m_o, r_f(m_m, m_w)) = m_o \cap (m_m \cup m_w) = m'$$

Since there are distinct terms (analysis trees) we can get distinct meanings.

Structural ambiguity, cont.

On the Saussurean account, we have rules F, G corresponding to f, g and μ for generating signs:

$$F(\langle e_1, m_1\rangle, \langle e_2, m_2\rangle) = \langle e_1 \; and \; e_2, m_1 \cup m_2\rangle$$
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$$F(G(\langle \textit{old}, m_o\rangle, \langle \textit{men}, m_m\rangle), \langle \textit{women}, m_w\rangle) = \langle \textit{old men and women}, m\rangle$$

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Conclusion: In the sign-based format, terms (analysis trees) are not needed to account for lexical or structural ambiguity; it suffices with rules that apply to pairs.

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(These are called ‘non-compositional idioms’ in Nunberg, Sag, and Wasow’s classical 1994 paper; there are various tests by which they can be recognized, e.g. they do not passivize.)

Let’s agree that the phrase *kick the bucket* has the following characteristics:

kick the bucket 1

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So the same rule ($f^{\mathbf{E}}$) generates the idiomatic *kick the bucket*, the language has the same expressions as before, but different terms, and

$$\mu(f(\textit{kick}, \textit{the bucket})) \neq \mu(f_I(\textit{kick}, \textit{the bucket}))$$

(where $r_{f_I}(m_0, m_1) = \mathrm{DIE}$).

kick the bucket 2

Can the pair format handle this without the use of terms?

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Yes. If *kick the bucket* is ambiguous, we have two signs

$$\langle \mathit{kick\ the\ bucket}, m\rangle \text{ and } \langle \mathit{kick\ the\ bucket}, \mathrm{DIE}\rangle$$

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Learning the idiom is simply learning the new rule $f_I^{\mathbf{L}}$.

Two algebraic formats

We saw that the classical set-up can be viewed as a syntactic algebra

$$\mathbf{E} = (E, A, f^{\mathbf{E}})_{f \in \Sigma}$$

generating E from the atoms in $A \subseteq E$ via the functions $f^{\mathbf{E}}$, and the semantics as a partial function

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Similarly, a sign-based setting consists of a pair grammar

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The grammar functions take care of both syntax and semantics ('in tandem').

Compositionality in the classical setting

Given $\mathbf{E}$, a semantics μ for $\mathbf{E}$ is compositional if

Funct(μ) for each $f \in \Sigma$ there is an operation r_f such that if $f(t_1, \ldots, t_n) \in dom(\mu)$, then $\mu(f(t_1, \ldots, t_n)) = r_f(\mu(t_1), \ldots, \mu(t_n))$

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Equivalently, letting

$$t \equiv_\mu u \text{ iff } \mu(t), \mu(u) \text{ are both defined and } \mu(t) = \mu(u)$$

we have (provided subterms of meaningful terms are always meaningful) the substitution version:

Subst($\equiv_\mu$) If $s[t_1, \ldots, t_k]$ and $s[u_1, \ldots, u_k]$ are both in $dom(\mu)$, and $t_i \equiv_\mu u_i$ for $i = 1, \ldots, n$, then $s[t_1, \ldots, t_k] \equiv_\mu s[u_1, \ldots, u_k]$

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(Here $t_1, \ldots, t_k$ are disjoint subterm occurrences in s.)

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(ii) (Saussurean) The meaning of a complex sign is determined by the meanings of its immediate constituent signs and the mode of composition.

(ii) makes good sense: each sign has a unique meaning, constituency for signs is defined via the term algebra for $\mathbf{L} = (L, A_L, f^{\mathbf{L}})_{f\in\Delta}$, and the modes of composition are the grammar rules $f^{\mathbf{L}}$.

Three kinds of Saussurean compositionality

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No. This is built into the sign-based format, where complex signs—and hence their meanings—are determined via the grammar rules by (the expressions and meanings of) their immediate constituent signs.

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Rec(μ) for each $f \in \Sigma$ there is an operation s_f such that if $f(t_1, \ldots, t_n) \in dom(\mu)$, then

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In other words: once you have a sign-based grammar, or a standard grammar + semantics, it is trivial in both cases that the meanings of complex expressions are determined (not computable!) by the meanings of their immediate parts and the parts themselves (and the mode of composition).

Saussurean compositionality: precise versions

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$\mathbf{L}$ is compositional iff for each $f \in \Delta$ there is an operation r_{2f} such that for $\langle e_1, m_1\rangle, \ldots, \langle e_n, m_n\rangle \in L$, if $f^{\mathbf{L}}(\langle e_1, m_1\rangle, \ldots, \langle e_n, m_n\rangle)$ is defined, then

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$\mathbf{L}$ is autonomous iff for each $f \in \Delta$ there is an operation r_{1f} such that for $\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle \in L$, if $f^{\mathbf{L}}(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle)$ is defined, then

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otherwise $r_{1f}(e_1, \ldots, e_n)$ or $r_{2f}(m_1, \ldots, m_n)$ is undefined.

A substitution version of Saussurean compositionality

Definition

$\mathbf{L}$ is right-centered if for all $f \in \Delta$, whenever $f^{\mathbf{L}}(\langle e_1, m_1\rangle, \ldots, \langle e_n, m_n\rangle)$ is defined and $\langle e_1', m_1\rangle, \ldots, \langle e_n', m_n\rangle \in L$, $f^{\mathbf{L}}(\langle e_1', m_1\rangle, \ldots, \langle e_n', m_n\rangle)$ is also defined.

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Fact

$\mathbf{L}$ *is compositional iff it is right-centered and Subst($\equiv^{2,\mathbf{L}}$) holds (similarly for autonomy and independence).*

Application: trivial Saussurean compositionality

Recall that in the classical format, if the meaning function μ is one-one, i.e. if there are no non-trivial synonymies, then μ is trivially compositional (use the substitution version).

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Fact

If L has no non-trivial synonymies (ambiguities), then any pair grammar for L is compositional (autonomous).

Notational variants?

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And vice versa, if starting from a grammar $\mathbf{E} = (E, A, f^{\mathbf{E}})_{f \in \Sigma}$ and a semantics μ for $\mathbf{E}$ with values in M, we could always find a pair grammar $\mathbf{L}_{\mathbf{E},\mu}$ such that conditions corresponding to (a) and (b) hold.

Notational variants?

Let $\mathbf{L} = (L, A_L, f^{\mathbf{L}})_{f\in\Delta}$ be a pair grammar, where $L \subseteq E \times M$. If there was a canonical way to associate a classical grammar $\mathbf{E_L} = (E, A, f^{\mathbf{E}})_{f\in\Delta}$ and a semantics $\mu_{\mathbf{L}}$ for $\mathbf{E_L}$ with values in M such that

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The two main problems are:

- the different treatments of ambiguity;
- translation works only when compositionality is assumed

From pair grammars to classical grammars + semantics 1

Let $\mathbf{L} = (L, A_L, f^{\mathbf{L}})_{f \in \Delta}$ be an independent pair grammar, so that for each $f \in \Delta$, the composition operations r_{1f} and r_{2f} are given.

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Thus, $a \in AE_{\mathbf{L}}$ has a unique meaning $\mu_{\mathbf{L}}(a) \in M$. When forming $GT_{\mathbf{E_L}}$ we can identify the atomic terms with the atomic expressions, and define the *complex* (grammatical) terms in $GT_{\mathbf{E_L}}$ as usual via the r_{1f}, simultaneously with the surjective homomorphism, call it val^E, from terms to E.

From pair grammars to classical grammars + semantics 2

Finally, we inductively extend $\mu_{\mathbf{L}}$ to (some) terms t in $GT_{\mathbf{E_L}}$, s.t. that for each t in $dom(\mu_{\mathbf{L}})$, $\langle val^E(t), \mu_{\mathbf{L}}(t) \rangle \in L$:

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Proof.

For $f \in \Delta$, the composition operation r_f is r_{2f}. $\square$

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Essentially, the result says that if we start with a sign-based grammar and want to turn it into the classical format, we shall only succeed if the sign-based grammar is independent.

Also, we must start by introducing new expressions ($bank_1$, $bank_2$, ...) to eliminate lexical ambiguity.

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Now let a grammar $\mathbf{E} = (E, A, f^{\mathbf{E}})_{f \in \Sigma}$ be given, with its corresponding set $GT_{\mathbf{E}}$ of grammatical terms and surjective homomorphism $val\colon GT_{\mathbf{E}} \to E$, and a compositional semantics μ for $\mathbf{E}$ with values in M.

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For $f \in \Sigma$, define a partial function $f^{\mathbf{E},\mu}$ from $(L_{\mathbf{E},\mu})^n$ to $L_{\mathbf{E},\mu}$: given $\langle val(t_1), \mu(t_1)\rangle, \ldots, \langle val(t_n), \mu(t_n)\rangle$ in $L_{\mathbf{E},\mu}$, let

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(b) *If in addition $(\mathbf{E}, \mu)$ has no lexical ambiguity (so $\overline{A} = A$), then, applying the previous construction to $\mathbf{L}_{\mathbf{E},\mu}$, we get back what we started from, i.e. $\mathbf{E}_{\mathbf{L}_{\mathbf{E},\mu}} = \mathbf{E}$ and $\mu_{\mathbf{L}_{\mathbf{E},\mu}} = \mu$.*

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We could conclude that, modulo ambiguity issues, the independent Saussurean pair grammar is indeed a notational variant (but in a rather weaker sense than I used before) of a classical style grammar and a compositional semantics.

Classical grammars + semantics vs. pair grammars

The correspondence seems to be something like this (disregarding issues of ambiguity and synonymy):

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$$\langle e, X, m\rangle \qquad (X \text{ is a syntactic category})$$

with the quotation rule as a unary function on triples

$$(q) \quad q^{\mathbf{L}}(\langle e, X, m\rangle) = \langle e, X, \langle e, X, m\rangle\rangle$$

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Third, however, it is clear that the grammar **L**, although autonomous, is not compositional in our sense: precisely because of the rule $q^{\mathbf{L}}$, substituting (terms denoting) triples with the same meaning does not preserve meaning.

This illustrates the unclarity of the 'in tandem' idea: if it just means the pair (or triple) format, it says nothing about compositionality.

Potts on quotation, cont.

But Potts could also turn the tables around and say: So what if it's not compositional—it's still a good grammar!

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Although the existence of suitable composition functions can now be trivial, their computability is not.

Conclusions

Some pro:s of pair grammars:

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The last point might just be a matter of taste though.

So far the only case of a realistic and fully specified computable but non-compositional grammar I have seen in the literature is the example by Potts just mentioned.

Further directions

NB The standard arguments for compositional semantics, in terms of learnability, productivity, systematicity, etc., show (at most) that computable (recursive) semantics is required.

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TO DO: look at

- complexity results for general compositionality;
- evaluate the pair format vs. the standard format from this perspective.

THANK YOU