Monotonicity in Aristotle's Modal Syllogistic

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1. Aristotle's Modal Syllogistic (*Prior Analytics* 1.3, 1.8–22)

- assertoric propositions:

AaB	(A belongs to all B; i.e. Every B is an A)
AeB	(A belongs to no B)
AiB	(A belongs to some B)
AoB	(A does not belong to some B)

- necessity propositions:

$\mathrm{Aa_N B}$	(A necessarily belongs to all B)
$\mathrm{Ae_N B}$	(A necessarily belongs to no B)
$\mathrm{Ai_N B}$	(A necessarily belongs to some B)
$\mathrm{Ao_N B}$	(A necessarily does not belong to some B)

T1: I call a term that into which the proposition is resolved, i.e. both the predicate and that of which it is predicated, 'is' or 'is not' being added. (*Prior Analytics* 1.1 24b16–18)

- Both assertoric and necessity propositions have a tripartite syntax:

predicate	copula	subject
A	belongs to all	B
A	necessarily belongs to all	B
A	necessarily does not belong to some	B

T2: (Necessity propositions) differ (from assertoric propositions) in that 'necessarily belongs' or 'necessarily does not belong' has been added to the terms. (*Prior Analytics* 1.8 29b39–30a2)

- Modal qualifications such as 'necessarily' are not sentential operators. They are part of a modally qualified copula which stands for a relation between terms.

- Aristotle holds that Barbara NXN is valid (*Prior Analytics* 1.9 30a15–23):

major premise:	$\mathrm{Aa_N B}$	(A necessarily belongs to all B)
minor premise:	BaC	(B belongs to all C)
conclusion:	$\mathrm{Aa_N C}$	(A necessarily belongs to all C)

- Theophrastus rejects Barbara NXN (Alexander *in Prior Analytics* 124.8–30):

	Animal necessarily belongs to all man
	Man belongs to all moving
but not:	Animal necessarily belongs to all moving
	Moving by means of legs necessarily belongs to all walking
	Walking belongs to all man
but not:	Moving by means of legs necessarily belongs to all man

- Aristotle holds that Celarent NXN is valid (*Prior Analytics* 1.9 30a15–23):

major premise:	$\mathrm{Ae_N B}$	(A necessarily belongs to no B)
minor premise:	BaC	(B belongs to all C)
conclusion:	$\mathrm{Ae_N C}$	(A necessarily belongs to no C)

- Aristotle also holds that the conversion of $\mathrm{e_N}$-propositions is valid (*Prior Analytics* 1.3 25a27–32):

premise:	$\mathrm{Ae_N B}$	(A necessarily belongs to no B)
conclusion:	$\mathrm{Be_N A}$	(B necessarily belongs to no A)

- If modality is analyzed in terms of modal sentential operators, there are two natural interpretations of $\mathrm{e_N}$-propositions:

de dicto:	$\mathrm{Ae_N B}$	if and only if	$\Box\forall x(\mathrm{B}x \supset \neg\mathrm{A}x)$
de re:	$\mathrm{Ae_N B}$	if and only if	$\forall x(\mathrm{B}x \supset \Box\neg\mathrm{A}x)$

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	Celarent NXN	conversion of $\mathrm{e_N}$-propositions
de dicto	invalid	valid
de re	valid	invalid

- It is often thought that Aristotle's $\mathrm{e_N}$-proposition are ambiguous: Celarent NXN requires a *de re* reading, whereas the rule of $\mathrm{e_N}$-conversion requires a *de dicto* reading: Becker (1933: 42), Kneale & Kneale (1962: 89–91), Hintikka (1973: 139–40), Sorabji (1980: 202), Mignucci (1998: 50–2 and 612), Striker (2009: xvi–xvii and 115). Similarly Patterson (1995: 41–87).

2. Monotonicity properties of assertoric propositions and necessity propositions

- Aristotle's claims of validity and invalidity in the apodeictic syllogistic, in *Prior Analytics* 1.3 and 1.8–12 (boldface indicates validity asserted by Aristotle, italics indicate invalidity asserted by him):

conversion from	to	
AaB	BiA	**25a17**
AiB	BiA	**25a20**
AeB	BeA	**25a15**
AoB	BoA	*25a22*
$\mathrm{Aa_N B}$	$\mathrm{Bi_N A}$	**25a32**
$\mathrm{Ai_N B}$	$\mathrm{Bi_N A}$	**25a32**
$\mathrm{Ae_N B}$	$\mathrm{Be_N A}$	**25a29**
$\mathrm{Ao_N B}$	$\mathrm{Bo_N A}$	*25a34*

first figure		XXX	NNN	NXN	XNN
Barbara:	AaB BaC ⊢ AaC	**25b37**	**29b36**	**30a17**	*30a23*
Celarent:	AeB BaC ⊢ AeC	**25b40**	**29b36**	**30a17**	*30a32*
Darii:	AaB BiC ⊢ AiC	**26a23**	**29b36**	**30a37**	*30b2*
Ferio:	AeB BiC ⊢ AoC	**26a25**	**29b36**	**30b1**	*30b5*

second figure		XXX	NNN	NXN	XNN
Cesare:	BeA BaC ⊢AeC	**27a5**	**29b36**	**30b9**	*30b18*
Camestres:	BaA BeC ⊢AeC	**27a9**	**29b36**	*30b20*	**30b14**
Festino:	BeA BiC ⊢ AoC	**27a32**	**29b36**	**31a5**	*invalid*
Baroco:	BaA BoC ⊢ AoC	**27a36**	**30a6**	*31a10*	*31a15*

third figure		XXX	NNN	NXN	XNN
Darapti:	AaB CaB ⊢ AiC	**28a17**	**29b36**	**31a24**	**31a31**
Felapton:	AeB CaB ⊢ AoC	**28a26**	**29b36**	**31a33**	*31a37*
Disamis:	AiB CaB ⊢ AiC	**28b7**	**29b36**	*31b31*	**31b12**
Datisi:	AaB CiB ⊢ AiC	**28b11**	**29b36**	**31b19**	*31b20*
Bocardo:	AoB CaB ⊢ AoC	**28b15**	**30a7**	*32a4*	*31b40*
Ferison:	AeB CiB ⊢ AoC	**28b31**	**29b36**	**31b33**	*32a1*

- Monotonicity of assertoric propositions with respect to a-predication (see van Eijck 1985: 16–17, van Benthem 1986: 111, Hodges 1998: 226–7):

AaB	AiB	AeB	AoB
↑↓	↑↑	↓↓	↓↑
Barbara	Darii	Camestres	Baroco
Barbara	Disamis	Celarent	Bocardo

Monotonicity with respect to a-predication

- Monotonicity of N-propositions with respect to a_N-predication (*Pr. An.* 1.8):

Aa_NB	Ai_NB	Ae_NB	Ao_NB
↑↓	↑↑	↓↓	↓↑
Barbara NNN	Darii NNN	Camestres NNN	Baroco NNN
Barbara NNN	Disamis NNN	Celarent NNN	Bocardo NNN

Monotonicity with respect to a_N-predication

- Monotonicity of N-propositions with respect to a-predication (1.9-11):

Aa_NB	Ai_NB	Ae_NB	Ao_NB
⋆↓	⋆⋆	↓↓	⋆⋆
Barbara XNN	*Darii XNN*	Camestres XNN	*Baroco XNN*
Barbara NXN	*Disamis NXN*	Celarent NXN	*Bocardo NXN*

Monotonicity with respect to a-predication

- Monotonicity is lost in most positions, but it is retained in three: in the subject term of a_N-propositions, and in the subject and predicate term of e_N-propositions. These three positions are downward monotonic.

3. Explaining the apodeictic syllogistic (*Prior Analytics* 1.3, 1.8-12)

- Explaining the apodeictic syllogistic by explaining the monotonicity properties of N-propositions with respect to a-predication: in principle, N-propositions are monotonic only with respect to a_N-predication, not with respect to a-predication. But in some positions, the latter kind of monotonicity is equivalent to the former. These are downward monotonic positions in which terms are required to be *per se* terms, i.e. terms which are a_N-predicated of everything of which they are a-predicated.

Thesis 1: If A is a *per se* term and AaB, then Aa_NB.

Thesis 2: If Aa_NB, then B is a *per se* term.

Thesis 3: If Ae_NB, then B is a *per se* term.

- A sound and complete deductive system for Aristotle's apodeictic syllogistic consists of the following three kinds of rules (see Malink 2013: 223-6):
  (1) Barbara, Celarent, Darii, Ferio NXN and NNN
  (2) Bocardo NNN, Baroco NNN
  (3) the conversion rules for assertoric and N-propositions listed above

Thesis 4: The pure necessity syllogistic mirrors the assertoric syllogistic: a syllogism or conversion rule consisting exclusively of N-propositions is valid iff the corresponding assertoric syllogism or conversion rule is valid (*Pr. An.* 1.8).

- Theses 1-4 entail the validity of all deduction rules just mentioned, except Darii NXN and Ferio NXN. The latter two syllogisms can be obtained as follows:

Thesis 5: (Assertoric Ecthesis) If AiB, then there is a C such that AaC and BaC.

Thesis 6: If there is a C such that BaC and Aa_NC, then Ai_NB.

Thesis 7: If there is a C such that BaC and Ae_NC, then Ao_NB.

- Theses 1-7 give a complete account of the apodeictic syllogistic: every claim of validity made by Aristotle in the apodeictic syllogistic (*Prior Analytics* 1.8-11) is entailed by Theses 1-7. In addition,Theses 1-7 are also sound with respect to the apodeictic syllogistic (since they are valid in the adequate semantics for the apodictic syllogistic given, e.g., in Malink 2013). Thus, Theses 1-7 give a sound and complete account of the apodeictic syllogistic.

- The converse of Thesis 7 is not valid. Otherwise Baroco XNN and Bocardo NXN could be proved by ecthesis as follows (cf. Alexander of Aphrodisias *in APr.* 144.23–145.4 and 151.22–8, Henle 1949: 99n27, van Rijen 1989: 195, Thom 1993: 198, 1996: 133):

1.	Ba_XA	[major premise]
2.	Bo_NC	[minor premise]
3.	for some Z, Ca_XZ and Be_NZ	[from 2;by converse of Thesis 7]
4.	Ca_XD and Be_ND	[from 3; by existential instantiation]
5.	Ca_XD and Ae_ND	[from 1, 4; by Camestres XNN]
6.	Ao_NC	[from 5; by Felapton NXN]

1.	Ao_NB	[major premise]
2.	Ca_XB	[minor premise]
3.	for some Z, Ba_XZ and Ae_NZ	[from 1;by converse of Thesis 7]
4.	Ba_XD and Ae_ND	[from 3; by existential instantiation]
5.	Ca_XD and Ae_ND	[from 2, 4; by Barbara XXX]
6.	Ao_NC	[from 5; by Felapton NXN]

- Problem: how to account for Aristotle's proofs by ecthesis of Baroco NNN and Bocardo NNN without assuming the converse of Thesis 7?

T3: It is necessary for us to set out something to which each of the two terms [i.e., the predicate term of the o_N-premise of Baroco NNN and Bocardo NNN] does not belong, and produce the deduction about this. For it will be necessary in application to these; and if it is necessary of what is set out, then it will be necessary of something of that former term; for what is set out is just a certain 'that'. Each of these deductions occurs in its own figure. (*Prior Analytics* 1.8 30a9–14)

- The most natural way to construct these proofs by ecthesis is by using the converse of Thesis 7:

1.	Ba_NA	[major premise]
2.	Bo_NC	[minor premise]
3.	for some Z, Ca_XZ and Be_NZ	[from 2; by converse of THESIS 7]
4.	Ca_XD and Be_ND	[from 3; by existential instantiation]
5.	Ca_XD and Ae_ND	[from 1, 4; by Camestres NNN]
6.	Ao_NC	[from 5; by Felapton NXN]

1.	Ao_NB	[major premise]
2.	Ca_NB	[minor premise]
3.	for some Z, Ba_XZ and Ae_NZ	[from 1;by converse of THESIS 7]
4.	Ba_XD and Ae_ND	[from 3; by existential instantiation]
5.	Ca_ND and Ae_ND	[from 2, 4; by Barbara NXN]
6.	Ao_NC	[from 5; by Felapton NNN]

4. A semantics for the apodeictic syllogistic

- The semantics is based on two primitive binary relations between terms, a-predication and a_N-predication, governed by the following axioms:

AXIOM 1	AaA
AXIOM 2	$AaB \wedge BaC \supset AaC$
AXIOM 3	$Aa_NB \wedge Ba_NC \supset Aa_NC$
AXIOM 4	$Aa_NB \wedge BaC \supset Ba_NC$
AXIOM 5	$Aa_NB \supset AaB$

- AXIOM 4 is motivated by THESES 1 and 2 above. AXIOM 1 is needed to deal with the problem of existential import (e.g., to guarantee that AaB implies BiA). Note: AXIOM 1 could be replaced by the weaker claim that if AaB, then BaB. (This would allow us to endorse a version of THESIS 2 for a-predication: if AaB, then B is a *per se* term.)

- The interpretation of Aristotle's assertoric and N-propositions in the semantics is as follows:

AaB	AaB
AeB	$\neg\exists Z(BaZ \wedge AaZ)$
AiB	$\exists Z(BaZ \wedge AaZ)$
AoB	$\neg(AaB)$

Aa_NB	Aa_NB
Ae_NB	$AeB \ \wedge \ \exists Z(Za_NA) \ \wedge \ \exists Z(Za_NB)$
Ai_NB	$\exists Z((BaZ \wedge Aa_NZ) \ \vee \ (AaZ \wedge Ba_NZ))$
Ao_NB	$\exists Z((BaZ \wedge Ae_NZ) \vee$ $(Ba_NZ \ \wedge \ \exists Y(Aa_NY) \ \wedge \ \forall X(AaX \wedge \exists Y(Ya_NX) \supset Be_NX))$

- These definitions, in conjunction with Axioms 1-5, constitute an adequate semantics for Aristotle's apodeictic syllogistic (cf. Malink 2013: 286-325). In particular, the semantics validates Bocardo NNN and Baroco NNN, but not Bocardo NXN and Baroco XNN.

- According to the above definition of e_N-propositions, Ae_NB implies that both A and B are *per se* terms. This is motivated by Thesis 3 and e_N-conversion (which is motivated by Thesis 4).

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