Monotonicity in Aristotle’s Modal Syllogistic
Marko Malink (New York University)

1. ARISTOTLE’S MODAL SYLLOGISTIC (*Prior Analytics* 1.3, 1.8–22)

- **assertoric propositions:**  
  - AaB (A belongs to all B; i.e. Every B is an A)
  - AeB (A belongs to no B)
  - AiB (A belongs to some B)
  - AoB (A does not belong to some B)

- **necessity propositions:**  
  - AaN B (A necessarily belongs to all B)
  - AeN B (A necessarily belongs to no B)
  - AiN B (A necessarily belongs to some B)
  - AoN B (A necessarily does not belong to some B)

**T1:** I call a term that into which the proposition is resolved, i.e. both the predicate and that of which it is predicated, ‘is’ or ‘is not’ being added. (*Prior Analytics* 1.1 24b16–18)

- Both assertoric and necessity propositions have a tripartite syntax:

<table>
<thead>
<tr>
<th>predicate</th>
<th>copula</th>
<th>subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>belongs to all</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>necessarily belongs to all</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>necessarily does not belong to some</td>
<td>B</td>
</tr>
</tbody>
</table>

**T2:** (Necessity propositions) differ (from assertoric propositions) in that ‘necessarily belongs’ or ‘necessarily does not belong’ has been added to the terms. (*Prior Analytics* 1.8 29b39–30a2)

- Modal qualifications such as ‘necessarily’ are not sentential operators. They are part of a modally qualified copula which stands for a relation between terms.

- Aristotle holds that Barbara NXN is valid (*Prior Analytics* 1.9 30a15–23):

<table>
<thead>
<tr>
<th>major premise:</th>
<th>minor premise:</th>
<th>conclusion:</th>
</tr>
</thead>
<tbody>
<tr>
<td>AaN B (A necessarily belongs to all B)</td>
<td>BaC (B belongs to all C)</td>
<td>AaN C (A necessarily belongs to all C)</td>
</tr>
</tbody>
</table>
• Theophrastus rejects Barbara NXN (Alexander in Prior Analytics 124.8–30):
  Animal necessarily belongs to all man
  Man belongs to all moving
  but not: Animal necessarily belongs to all moving
  Moving by means of legs necessarily belongs to all walking
  Walking belongs to all man
  but not: Moving by means of legs necessarily belongs to all man

• Aristotle holds that Celarent NXN is valid (Prior Analytics 1.9 30a15–23):
  major premise: \( \text{Ae}_N \text{B} \) (A necessarily belongs to no B)
  minor premise: \( \text{BaC} \) (B belongs to all C)
  conclusion: \( \text{Ae}_N \text{C} \) (A necessarily belongs to no C)

• Aristotle also holds that the conversion of \( e_N \)-propositions is valid (Prior Analytics 1.3 25a27–32):
  premise: \( \text{Ae}_N \text{B} \) (A necessarily belongs to no B)
  conclusion: \( \text{Be}_N \text{A} \) (B necessarily belongs to no A)

• If modality is analyzed in terms of modal sentential operators, there are two natural interpretations of \( e_N \)-propositions:
  \textit{de dicto}: \( \text{Ae}_N \text{B} \) if and only if \( \square \forall x (Bx \supset \neg A_x) \)
  \textit{de re}: \( \text{Ae}_N \text{B} \) if and only if \( \forall x (Bx \supset \square \neg A_x) \)

• Celarent NXN conversion of \( e_N \)-propositions
  \begin{tabular}{l|ll}
  & Celarent NXN & conversion of \( e_N \)-propositions \\
  \textit{de dicto} & invalid & valid \\
  \textit{de re} & valid & invalid \\
  \end{tabular}

2. Monotonicity properties of assertoric propositions and necessity propositions

- Aristotle’s claims of validity and invalidity in the apodeictic syllogistic, in *Prior Analytics* 1.3 and 1.8–12 (boldface indicates validity asserted by Aristotle, italics indicate invalidity asserted by him):

<table>
<thead>
<tr>
<th>Conversion from</th>
<th>To</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>AaB</td>
<td>BiA</td>
<td>25a17</td>
</tr>
<tr>
<td>AiB</td>
<td>BiA</td>
<td>25a20</td>
</tr>
<tr>
<td>AeB</td>
<td>BeA</td>
<td>25a15</td>
</tr>
<tr>
<td>AoB</td>
<td>BoA</td>
<td>25a22</td>
</tr>
<tr>
<td>AaN B</td>
<td>BiN A</td>
<td>25a32</td>
</tr>
<tr>
<td>AiN B</td>
<td>BiN A</td>
<td>25a32</td>
</tr>
<tr>
<td>AeN B</td>
<td>BeN A</td>
<td>25a29</td>
</tr>
<tr>
<td>AoN B</td>
<td>BoN A</td>
<td>25a34</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>First figure</th>
<th>XXX</th>
<th>NNN</th>
<th>NXN</th>
<th>XNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara:</td>
<td>AaB BaC ⊢ AaC</td>
<td>25b37</td>
<td>29b36</td>
<td>30a17</td>
</tr>
<tr>
<td>Celarent:</td>
<td>AeB BaC ⊢ AeC</td>
<td>25b40</td>
<td>29b36</td>
<td>30a17</td>
</tr>
<tr>
<td>Darii:</td>
<td>AaB BiC ⊢ AiC</td>
<td>26a23</td>
<td>29b36</td>
<td>30a37</td>
</tr>
<tr>
<td>Ferio:</td>
<td>AeB BiC ⊢ AoC</td>
<td>26a25</td>
<td>29b36</td>
<td>30b1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>XXX</th>
<th>NNN</th>
<th>NXN</th>
<th>XNN</th>
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</thead>
<tbody>
<tr>
<td>Cesare:</td>
<td>BeA BaC ⊢ AeC</td>
<td>27a5</td>
<td>29b36</td>
<td>30b9</td>
</tr>
<tr>
<td>Camestres:</td>
<td>BaA BeC ⊢ AeC</td>
<td>27a9</td>
<td>29b36</td>
<td>30b20</td>
</tr>
<tr>
<td>Festino:</td>
<td>BeA BiC ⊢ AoC</td>
<td>27a32</td>
<td>29b36</td>
<td>31a5</td>
</tr>
<tr>
<td>Baroco:</td>
<td>BaA BoC ⊢ AoC</td>
<td>27a36</td>
<td>30a6</td>
<td>31a10</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>AaB</th>
<th>AiB</th>
<th>AeB</th>
<th>AoB</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑↓</td>
<td>↑↑</td>
<td>↓↓</td>
<td>↓↑</td>
</tr>
</tbody>
</table>

Barbara Darii Camestres Baroco
Barbara Disamis Celarent Bocardo

Monotonicity with respect to a-predication

Monotonicity of N-propositions with respect to aN-predication (Pr. An. 1.8):

<table>
<thead>
<tr>
<th>AaN B</th>
<th>AiN B</th>
<th>AeN B</th>
<th>AoN B</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑↓</td>
<td>↑↑</td>
<td>↓↓</td>
<td>↓↑</td>
</tr>
</tbody>
</table>

Barbara NNN Darii NNN Camestres NNN Baroco NNN
Barbara NNN Disamis NNN Celarent NNN Bocardo NNN

Monotonicity with respect to aN-predication

Monotonicity of N-propositions with respect to a-predication (1.9-11):

<table>
<thead>
<tr>
<th>AaN B</th>
<th>AiN B</th>
<th>AeN B</th>
<th>AoN B</th>
</tr>
</thead>
<tbody>
<tr>
<td>* ↓</td>
<td>**</td>
<td>↓↓</td>
<td>**</td>
</tr>
</tbody>
</table>

Barbara XNN Darii XNN Camestres XNN Baroco XNN
Barbara NXN Disamis NXN Celarent NXN Bocardo NXN

Monotonicity with respect to a-predication
• Monotonicity is lost in most positions, but it is retained in three: in the subject term of an\textsubscript{N}-propositions, and in the subject and predicate term of en\textsubscript{N}-propositions. These three positions are downward monotonic.

3. Explaining the apodeictic syllogistic (Prior Analytics 1.3, 1.8-12)

• Explaining the apodeictic syllogistic by explaining the monotonicity properties of N-propositions with respect to a-predication: in principle, N-propositions are monotonic only with respect to a\textsubscript{N}-predication, not with respect to a-predication. But in some positions, the latter kind of monotonicity is equivalent to the former. These are downward monotonic positions in which terms are required to be per se terms, i.e. terms which are a\textsubscript{N}-predicated of everything of which they are a-predicated.

Thesis 1: If A is a per se term and AaB, then Aa\textsubscript{N}B.

Thesis 2: If Aa\textsubscript{N}B, then B is a per se term.

Thesis 3: If Ae\textsubscript{N}B, then B is a per se term.

• A sound and complete deductive system for Aristotle’s apodeictic syllogistic consists of the following three kinds of rules (see Malink 2013: 223-6):

(1) Barbara, Celarent, Darii, Ferio NXN and NNN
(2) Bocardo NNN, Baroco NNN
(3) the conversion rules for assertoric and N-propositions listed above

Thesis 4: The pure necessity syllogistic mirrors the assertoric syllogistic: a syllogism or conversion rule consisting exclusively of N-propositions is valid iff the corresponding assertoric syllogism or conversion rule is valid (Pr. An. 1.8).

• Theses 1-4 entail the validity of all deduction rules just mentioned, except Darii NXN and Ferio NXN. The latter two syllogisms can be obtained as follows:

Thesis 5: (Assertoric Ecthesis) If AiB, then there is a C such that AaC and BaC.

Thesis 6: If there is a C such that BaC and Aa\textsubscript{N}C, then Ai\textsubscript{N}B.

Thesis 7: If there is a C such that BaC and Ae\textsubscript{N}C, then Ao\textsubscript{N}B.
• Theses 1-7 give a complete account of the apodeictic syllogistic: every claim of validity made by Aristotle in the apodeictic syllogistic (Prior Analytics 1.8-11) is entailed by Theses 1-7. In addition, Theses 1-7 are also sound with respect to the apodeictic syllogistic (since they are valid in the adequate semantics for the apodictic syllogistic given, e.g., in Malink 2013). Thus, Theses 1-7 give a sound and complete account of the apodeictic syllogistic.


1. BaXA [major premise]
2. BoNC [minor premise]
3. for some Z, CaXZ and BeNZ [from 2; by converse of Thesis 7]
4. CaXD and BeND [from 3; by existential instantiation]
5. CaXD and AeND [from 1, 4; by Camestres XNN]
6. AoNC [from 5; by Felapton NXN]

1. AoNB [major premise]
2. CaXB [minor premise]
3. for some Z, BaXZ and AeNZ [from 1; by converse of Thesis 7]
4. BaXD and AeND [from 3; by existential instantiation]
5. CaXD and AeND [from 2, 4; by Barbara XXX]
6. AoNC [from 5; by Felapton NXN]

• Problem: how to account for Aristotle’s proofs by ecthesis of Baroco NNN and Bocardo NNN without assuming the converse of Thesis 7?

T3: It is necessary for us to set out something to which each of the two terms [i.e., the predicate term of the oN-premise of Baroco NNN and Bocardo NNN] does not belong, and produce the deduction about this. For it will be necessary in application to these; and if it is necessary of what is set out, then it will be necessary of something of that former term; for what is set out is just a certain ‘that’. Each of these deductions occurs in its own figure. (Prior Analytics 1.8 30a9–14)

• The most natural way to construct these proofs by ecthesis is by using the converse of Thesis 7:
1. $Ba_N A$ [major premise]
2. $Bo_N C$ [minor premise]
3. for some Z, $Ca_X Z$ and $Be_N Z$ [from 2; by converse of Thesis 7]
4. $Ca_X D$ and $Be_N D$ [from 3; by existential instantiation]
5. $Ca_X D$ and $Ae_N D$ [from 1, 4; by Camestres NNN]
6. $Ao_N C$ [from 5; by Felapton NNN]

1. $Ao_N B$ [major premise]
2. $Ca_N B$ [minor premise]
3. for some Z, $Ba_X Z$ and $Ae_N Z$ [from 1; by converse of Thesis 7]
4. $Ba_X D$ and $Ae_N D$ [from 3; by existential instantiation]
5. $Ca_N D$ and $Ae_N D$ [from 2, 4; by Barbara NNN]
6. $Ao_N C$ [from 5; by Felapton NNN]

### 4. A SEMANTICS FOR THE APODEICTIC SYLLOGISTIC

- The semantics is based on two primitive binary relations between terms, $a$-predication and $a_N$-predication, governed by the following axioms:

<table>
<thead>
<tr>
<th>Axiom 1</th>
<th>$AaA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axiom 2</td>
<td>$AaB \land BaC \supset AaC$</td>
</tr>
<tr>
<td>Axiom 3</td>
<td>$Aa_N B \land Ba_N C \supset Aa_N C$</td>
</tr>
<tr>
<td>Axiom 4</td>
<td>$Aa_N B \land BaC \supset Ba_N C$</td>
</tr>
<tr>
<td>Axiom 5</td>
<td>$Aa_N B \supset AaB$</td>
</tr>
</tbody>
</table>

- **Axiom 4** is motivated by **Theses 1 and 2** above. **Axiom 1** is needed to deal with the problem of existential import (e.g., to guarantee that $AaB$ implies $BiA$). Note: **Axiom 1** could be replaced by the weaker claim that if $AaB$, then $BaB$. (This would allow us to endorse a version of **Thesis 2** for $a$-predication: if $AaB$, then $B$ is a *per se* term.)

- The interpretation of Aristotle’s assertoric and N-propositions in the semantics is as follows:

<table>
<thead>
<tr>
<th>$AaB$</th>
<th>$AaB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AeB$</td>
<td>$\neg \exists Z(BaZ \land AaZ)$</td>
</tr>
<tr>
<td>$AiB$</td>
<td>$\exists Z(BaZ \land AaZ)$</td>
</tr>
<tr>
<td>$AoB$</td>
<td>$\neg (AaB)$</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\text{Aa}_N B & \quad \text{Aa}_N B \\
\text{Ae}_N B & \quad \text{Ae} B \land \exists Z(Za_N A) \land \exists Z(Za_N B) \\
\text{Ai}_N B & \quad \exists Z((BaZ \land Aa_N Z) \lor (AaZ \land BaZ)) \\
\text{Ao}_N B & \quad \exists Z((BaZ \land \text{Ae}_N Z) \lor (BaZ \land \exists Y(AaN Y) \land \forall X(AaX \land \exists Y(YaX) \supset \text{Be}_N X))
\end{align*}
\]

- These definitions, in conjunction with Axioms 1-5, constitute an adequate semantics for Aristotle’s apodeictic syllogistic (cf. Malink 2013: 286-325). In particular, the semantics validates Bocardo NNN and Baroco NNN, but not Bocardo NXN and Baroco XNN.

- According to the above definition of e_N-propositions, Ae_N B implies that both A and B are per se terms. This is motivated by Thesis 3 and e_N-conversion (which is motivated by Thesis 4).

References


Henle, P. (1949): ‘On the Fourth Figure of the Syllogism’, *Philosophy of Science* 16, 94–104.


