CETERIS PARIBUS LOGIC IN COUNTERFACTUAL REASONING

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Ceteris Paribus, the absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies.
Nash Equilibrium

- A strategy profile in which none of the agents could deviate to their own advantage, ceteris paribus.

\[
\begin{array}{cccc}
  & a_1 & \vline & a_i & \vline & a_m \\
 b_i & (a_1, b_i) & \vline & & \vline & \\
 b_j & & \vline & u & \vline & \\
 b_n & & \vline & (a_m, b_n) & \vline & \\
\end{array}
\]
Counterfactuals

- If kangaroos had no tails (k), they would topple over (t).

- David Lewis (1973): in each of the most similar worlds where kangaroos have no tails, they topple over.
The Nixon Counterfactual
The Nixon Counterfactual

• Kit Fine (1975): If Nixon had pushed the button, there would have been a nuclear holocaust.
The Fine Model

\[ p = \text{“Nixon pushes the button,”} \]
\[ s = \text{“the missile successfully launches,”} \]
\[ m = \text{“a miracle prevents the missile being launched,”} \]
\[ h = \text{“a nuclear holocaust occurs,”} \]

\[ F, w \not\models p \rightarrow h \]
The Lewis Model

• It is of the first importance to avoid big, widespread, diverse violations of law.

• It is of the second importance to maximize the spatio-temporal region throughout which perfect match of particular fact prevails.

• It is of the third importance to avoid even small, localized, simple violations of law.

• It is of little or no importance to secure approximate similarity of particular fact, even in matters that concern us greatly.
The Lewis Model

\[ p = \text{“Nixon pushes the button,”} \]
\[ s = \text{“the missile successfully launches,”} \]
\[ m = \text{“a miracle prevents the missile being launched,”} \]
\[ h = \text{“a nuclear holocaust occurs,”} \]

\[ \mathcal{L}, w \models p \implies h \]
Ceteris Paribus Semantics

\[ w \equiv_\Gamma v \iff \forall \gamma \in \Gamma : M, w \models \gamma \iff M, v \models \gamma \]

- ψ is true in every most similar \( \varphi \)-world that keep all of \( \Gamma \) equal.

\[
\begin{align*}
[\varphi, \Gamma] & \psi \\
[\varphi, \emptyset] & \psi = \square \\
\end{align*}
\]
Everything else equal, not normal

Were I to strike the match, it would light.
Back to the Fine Model

\[ \mathcal{F}, w \models [p, \{m\}]h \]
Ceteris Paribus & Dynamics

Before $\mathcal{F}$

$\mathcal{F}$

$w$

$u$

$p, s, h$

$v$

$p, m$

After $[\{m\}]\mathcal{F}$

$\mathcal{F}$

$w$

$u$

$p, s, h$

$v$

$p, m$
Kit Fine (1975): If Nixon had pushed the button (p), there would have been a nuclear holocaust (h).

Let p and h vary (UD), keep everything else equal.

\[ [p, \{m, s\}]h \]
Universe of Discourse

\[
\begin{align*}
\text{UD}(p) &= \{p\} \\
\text{UD}(\neg \varphi) &= \text{UD}(\varphi) \\
\text{UD}(\varphi \lor \psi) &= \text{UD}(\varphi) \cup \text{UD}(\psi) \\
\text{UD}([\varphi, \Gamma]\psi) &= \text{UD}(\varphi) \cup \text{UD}(\Gamma) \cup \text{UD}(\psi) \\
\text{UD}(\{\gamma_1, \ldots, \gamma_n\}) &= \text{UD}(\gamma_1) \cup \cdots \cup \text{UD}(\gamma_n).
\end{align*}
\]

\[
[\varphi, \text{PROP} \setminus (\text{UD}(\varphi) \cup \text{UD}(\psi))]\psi
\]
Strict CP

Before $\mathcal{F}$

\[ \mathcal{F} \]

\[ w \]

\[ \cdots \]

\[ \mathcal{u} \]

\[ p, s, h \]

\[ \mathcal{v} \]

\[ p, m \]

After $[\{m, s\}]\mathcal{F}$

\[ \mathcal{F} \]

\[ w \]

\[ \mathcal{u} \]

\[ p, s, h \]

\[ \mathcal{v} \]

\[ p, m \]

\[ \mathcal{F}, w \models [p, \{m, s\}]h \]

\[ \mathcal{F}, w \models [p, \{m, s\}]\neg h \]
Relaxing the ceteris paribus clause
(Naive counting)

- $\varphi$ is true in every most similar $\varphi$-world that keeps $\Gamma$ equal as much as possible.

\[
A^M_{\Gamma}(u, v) = \{ \gamma \in \Gamma : \mathcal{M}, u \models \gamma \iff \mathcal{M}, v \models \gamma \} 
\]

$u \preceq^\Gamma_w v \iff$

either $|A^M_{\Gamma}(u, w)| > |A^M_{\Gamma}(v, w)|$,

or $|A^M_{\Gamma}(u, w)| = |A^M_{\Gamma}(v, w)|$ and $u \preceq_w v$. 
Naive Counting (NC)

\[ A^\mathcal{F}_{\{m,s\}}(w,u) = \{m\} \]
\[ A^\mathcal{F}_{\{m,s\}}(w,v) = \{s\} \]
\[ \mathcal{F}, w \models [p, \{m, s\}] - h \]
Relaxing the ceteris paribus clause  
(Maximal supersets)

- $\varphi$ is true in every most similar $\varphi$-world that keeps $\Gamma$ equal as much as possible.

$$A^\mathcal{M}_\Gamma(u, v) = \{ \gamma \in \Gamma : \mathcal{M}, u \models \gamma \text{ iff } \mathcal{M}, v \models \gamma \}$$

$u \unlhd^\Gamma_w v$ iff

either $A^\mathcal{M}_\Gamma(v, w) \subset A^\mathcal{M}_\Gamma(u, w)$,

or $A^\mathcal{M}_\Gamma(v, w) = A^\mathcal{M}_\Gamma(u, w)$ and $u \leq^w v$. 
Before $\mathcal{F}$

\[ \mathcal{F} \]

$w$ \rightarrow $u$ \rightarrow $v$

$p, s, h$

$v$ \rightarrow $p, m$

After $\left\{ \{m, s\} \right\} \mathcal{F}$

\[ \mathcal{F} \]

$w$ \rightarrow $u$ \rightarrow $v$

$p, s, h$

$v$ \rightarrow $p, m$

\[
A_{\mathcal{F}}^{\{m,s\}}(w,u) = \{m\} \\
A_{\mathcal{F}}^{\{m,s\}}(w,v) = \{s\}
\]

$\mathcal{F}, w \nvdash [p, \{m, s\}]h$
Comparative Possibility

\[ \varphi \preceq \psi \quad = \quad \varphi \text{ is at least as possible as } \psi. \]

i.e., each \( \psi \)-world can be matched by a \textit{better} \( \varphi \)-world.

\[ \varphi < \psi \quad := \quad \neg (\psi \preceq \varphi) \]

\[ \Diamond \varphi \quad := \quad \varphi < \bot \]

\[ \varphi \square \rightarrow \psi \quad := \quad \Diamond \varphi \rightarrow (\varphi \land \psi) < (\varphi \land \neg \psi) \]
Comparative Similarity, CP

\[ u \equiv \Gamma v \quad \text{iff} \quad \forall \gamma \in \Gamma : M, u \models \gamma \quad \text{iff} \quad M, v \models \gamma \]

\[ [w]_{\Gamma} = \{ u \in W_w : w \equiv \Gamma u \} \]

\[ \triangleleft_{w} \overset{\Gamma}{=} = \leq_w \cap ([w]_{\Gamma} \times [w]_{\Gamma}) \]

\[ \varphi \triangleleft_{\Gamma} \psi \quad = \quad \varphi \quad \text{is at least as possible as} \quad \psi, \quad \text{keeping} \quad \Gamma \quad \text{equal} \]
Completeness

- Strict CP:

\[ \varphi \preceq^\Gamma \psi \iff \bigwedge_{\Gamma \in \mathfrak{P}} \left[ \Gamma \rightarrow (\varphi \land \bar{\Gamma}) \preceq (\psi \land \bar{\Gamma}) \right] \]

- Naive Counting

\[ \varphi \preceq^\Gamma \psi \iff \bigwedge_{\Gamma \in \mathfrak{P}} \left( \bar{\Gamma} \rightarrow \bigwedge_{\Lambda \subseteq \bar{\Gamma}} \left[ \left( (\varphi \land \Lambda) \lor \bigvee_{|\Lambda|<|\Sigma|\leq|\bar{\Gamma}|} \Sigma \land \varphi \right) \preceq \psi \land \Lambda \right] \right) \]

- Maximal Supersets

\[ \varphi \sqsupseteq^\Gamma \psi \iff \bigwedge_{\Gamma \in \mathfrak{P}} \left( \bar{\Gamma} \rightarrow \bigwedge_{\Lambda \subseteq \bar{\Gamma}} \left[ \left( (\varphi \land \Lambda) \lor \bigvee_{\Lambda \subset \Sigma \subseteq \bar{\Gamma}} \Sigma \land \varphi \right) \preceq \psi \land \Lambda \right] \right) \]
Future Work

- Indeterminism.
- Most Counterfactuals are false (Alan Hájek)
- Infinite CP sets.
- Applications in game theory?