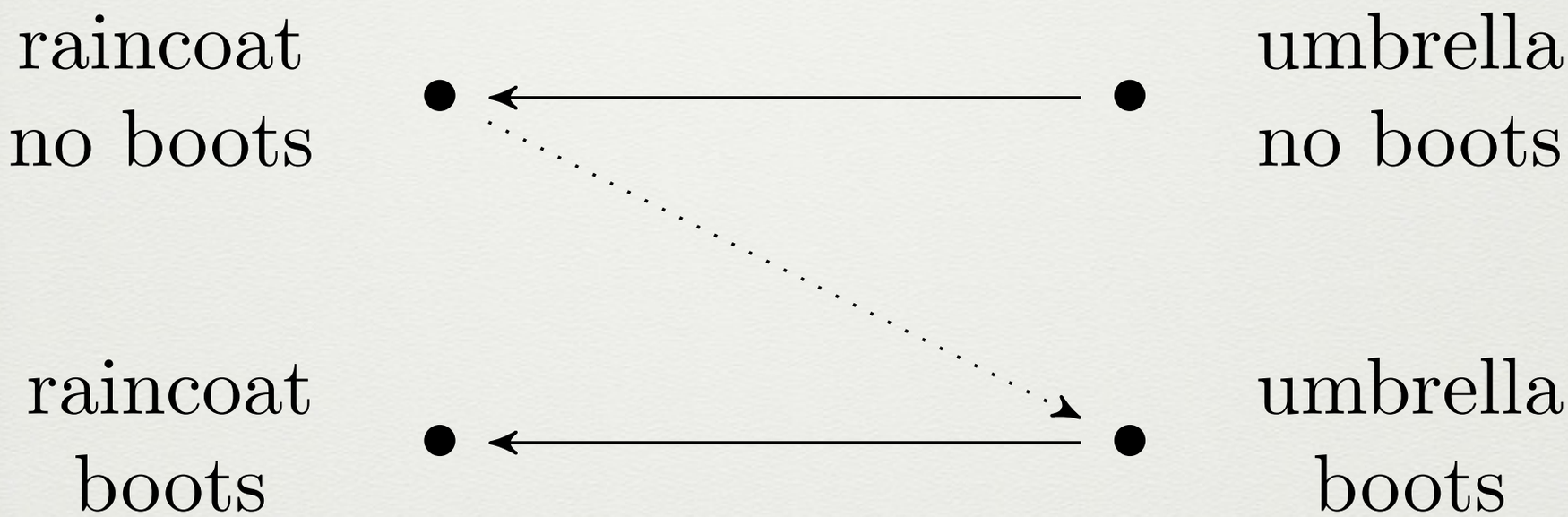
CETERIS PARIBUS LOGIC IN COUNTERFACTUAL REASONING

Patrick Girard & Marcus Anthony Triplett
University of Auckland

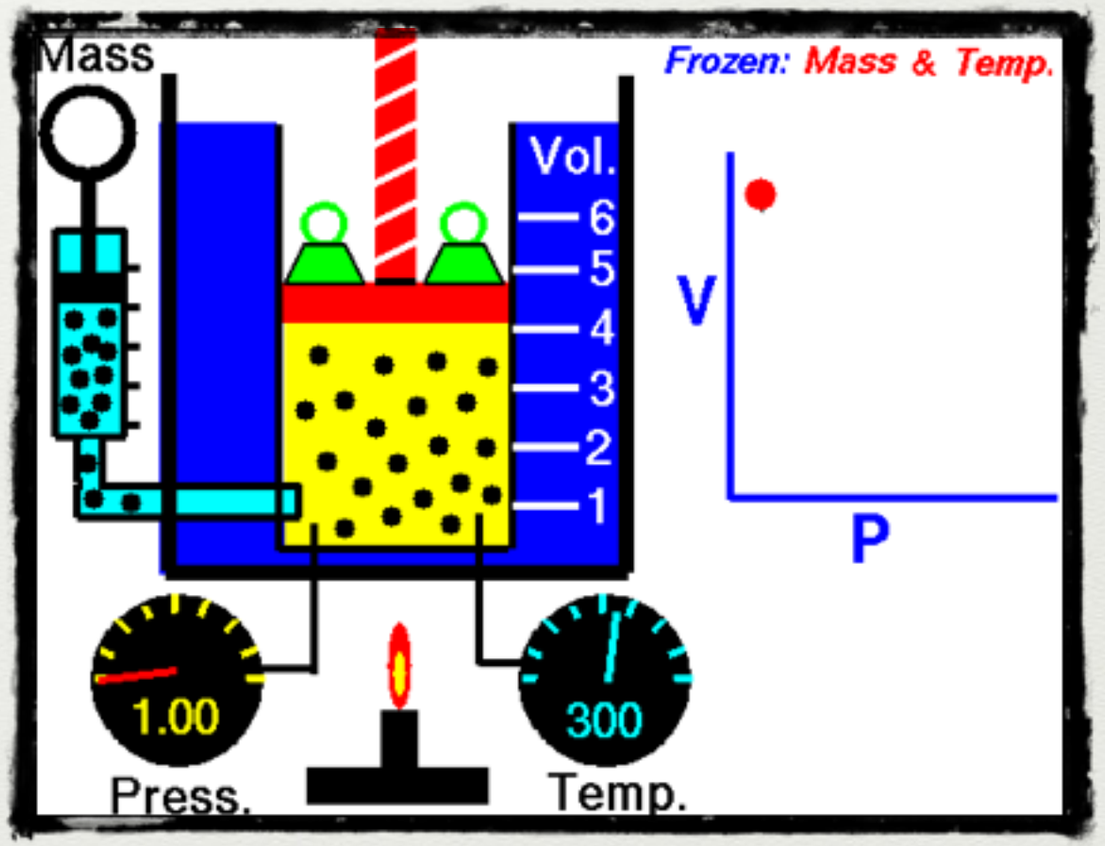
Preference Logic



Georg H. von Wright (1963): Ceteris paribus, I prefer my raincoat to my umbrella.

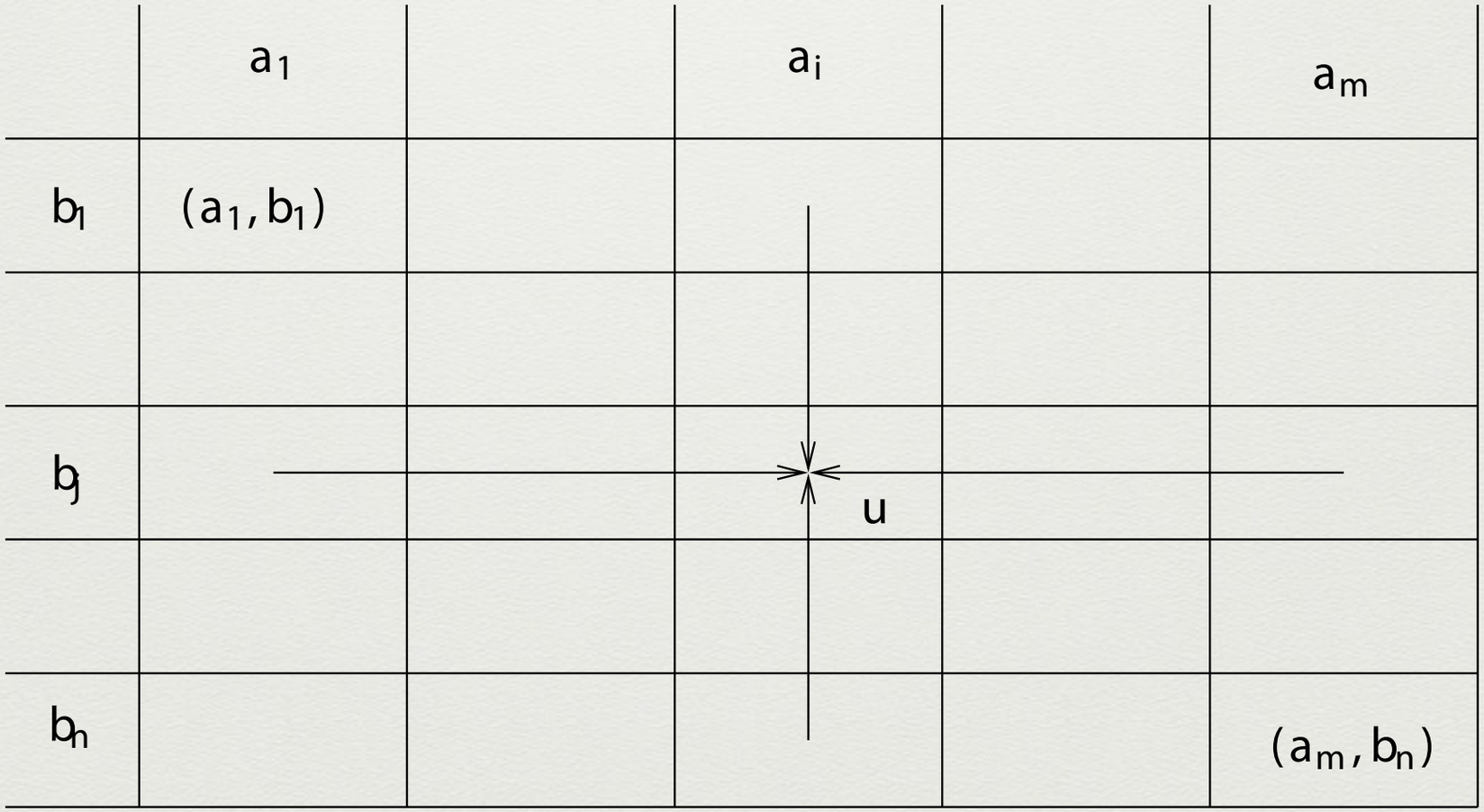
Boyle's Law

- Ceteris Paribus, the absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies.



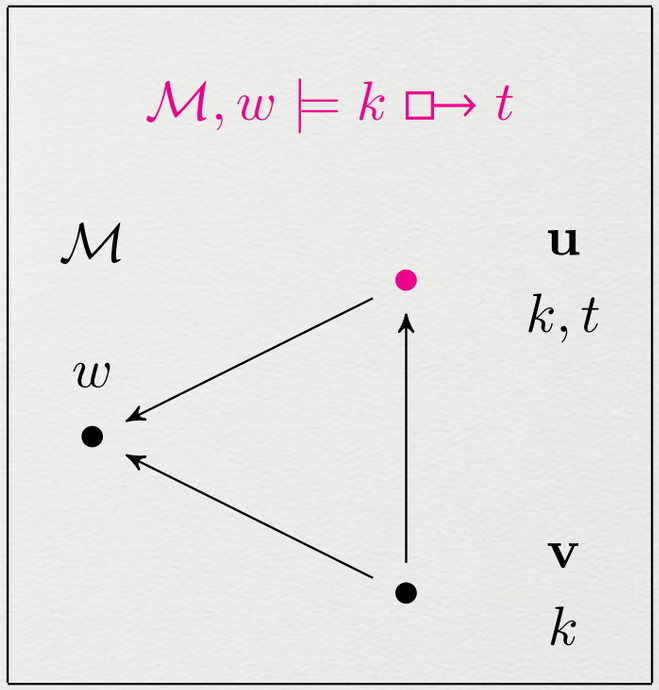
Nash Equilibrium

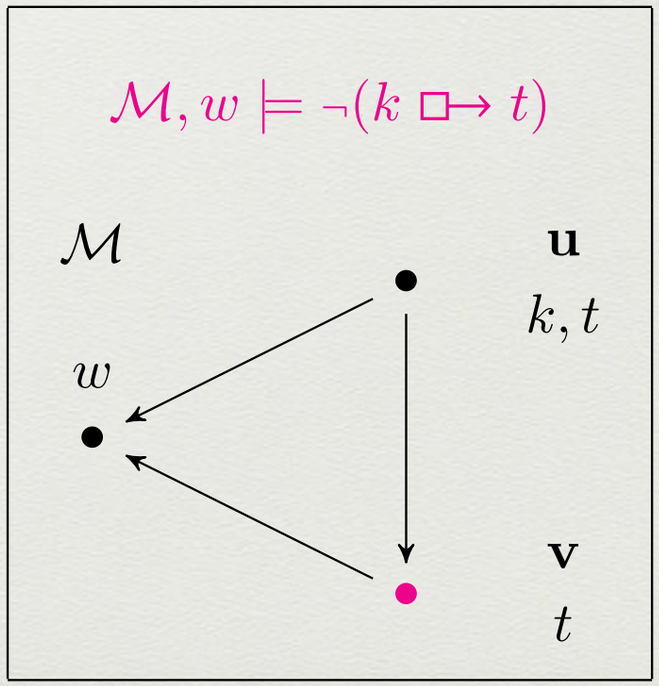
- A strategy profile in which none of the agents could deviate to their own advantage, ceteris paribus.



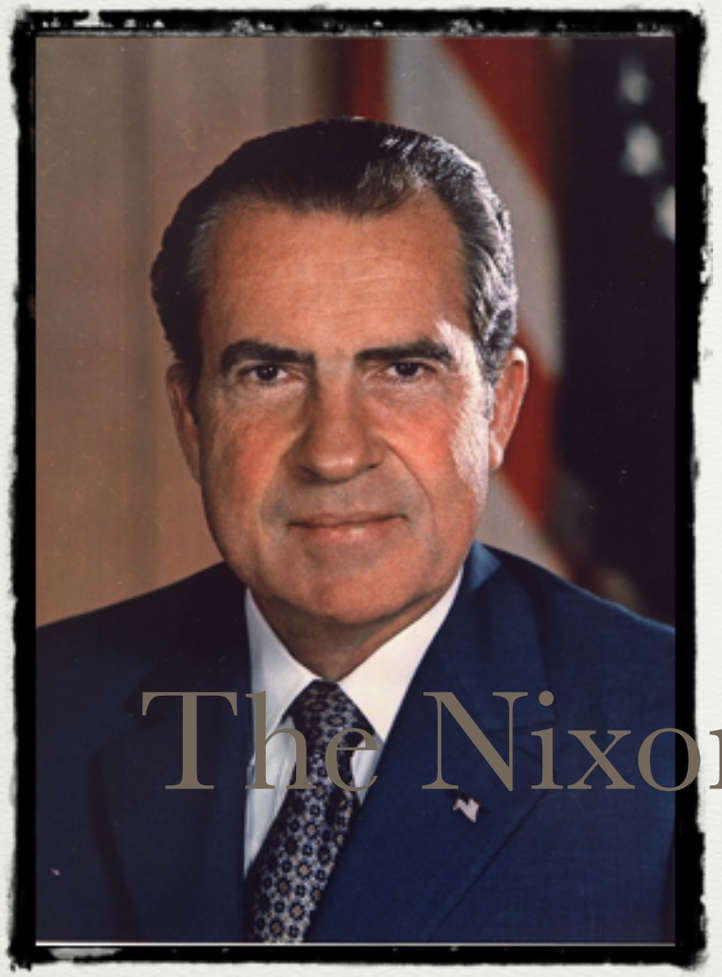
Counterfactuals

- If kangaroos had no tails (k), they would topple over (t).
- David Lewis (1973): in each of the most similar worlds where kangaroos have no tails, they topple over.





The Nixon Counterfactual

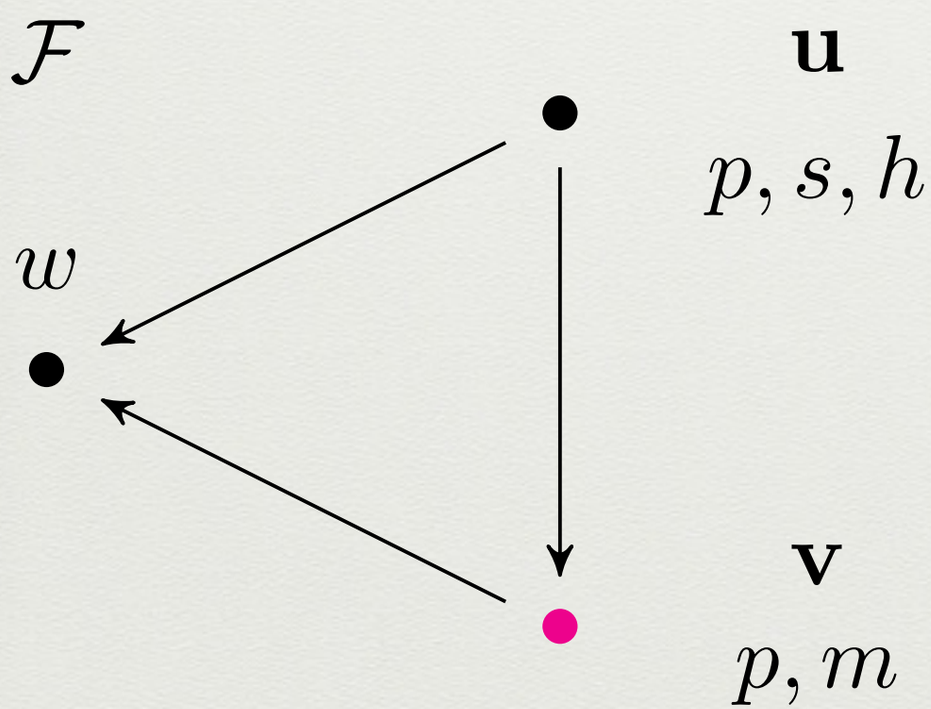


The Nixon Counterfactual

- Kit Fine (1975): If Nixon had pushed the button, there would have been a nuclear holocaust.

The Fine Model

p = "Nixon pushes the button,"
s = "the missile successfully launches,"
m = "a miracle prevents the missile being launched,"
h = "a nuclear holocaust occurs,"



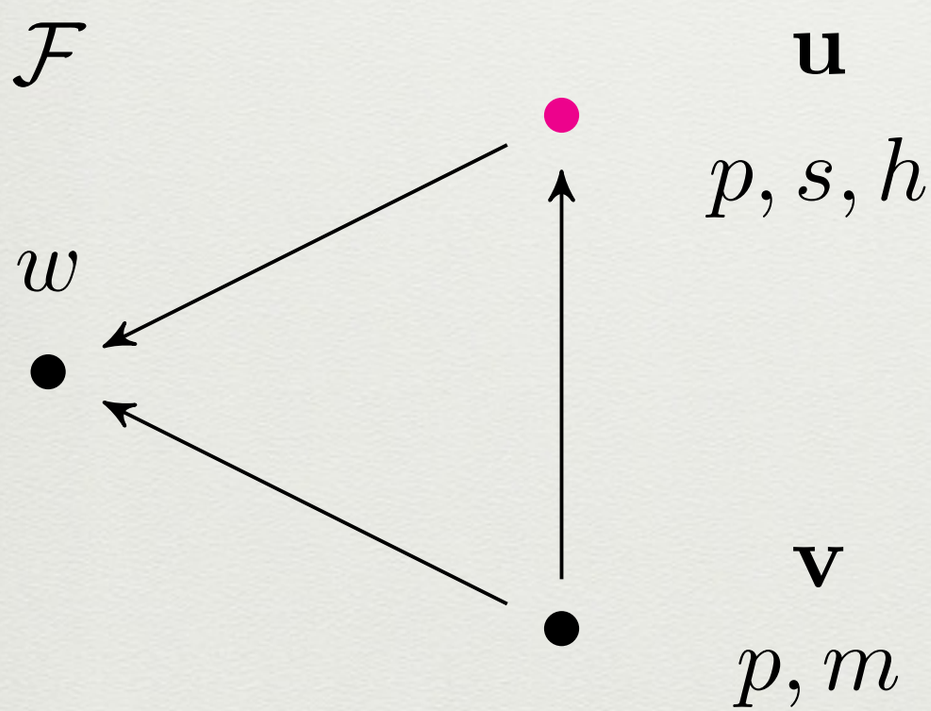
$$\mathcal{F}, w \not\models p \mathrel{\Box\!\!\rightarrow} h$$

The Lewis Model

- It is of the first importance to avoid big, widespread, diverse violations of law.
- It is of the second importance to maximize the spatio-temporal region throughout which perfect match of particular fact prevails.
- It is of the third importance to avoid even small, localized, simple violations of law.
- It is of little or no importance to secure approximate similarity of particular fact, even in matters that concern us greatly.

The Lewis Model

p = "Nixon pushes the button,"
s = "the missile successfully launches,"
m = "a miracle prevents the missile being launched,"
h = "a nuclear holocaust occurs,"



$$\mathcal{L}, w \models p \mathrel{\Box\!\!\rightarrow} h$$

Ceteris Paribus Semantics

$$w \equiv_\Gamma v \text{ iff } \forall \gamma \in \Gamma : M, w \models \gamma \text{ iff } M, v \models \gamma$$

- ψ is true in every most similar φ-world that keep all of Γ equal.

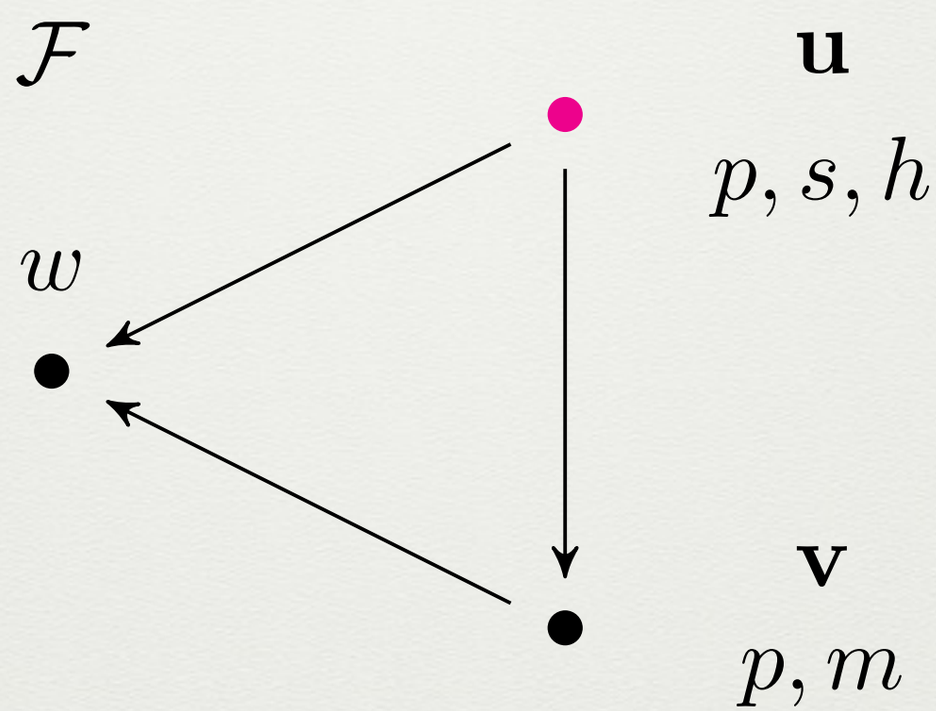
$$[\varphi, \Gamma]\psi$$

$$[\varphi, \emptyset]\psi = \Box\!\!\rightarrow$$

Everything else equal, not normal

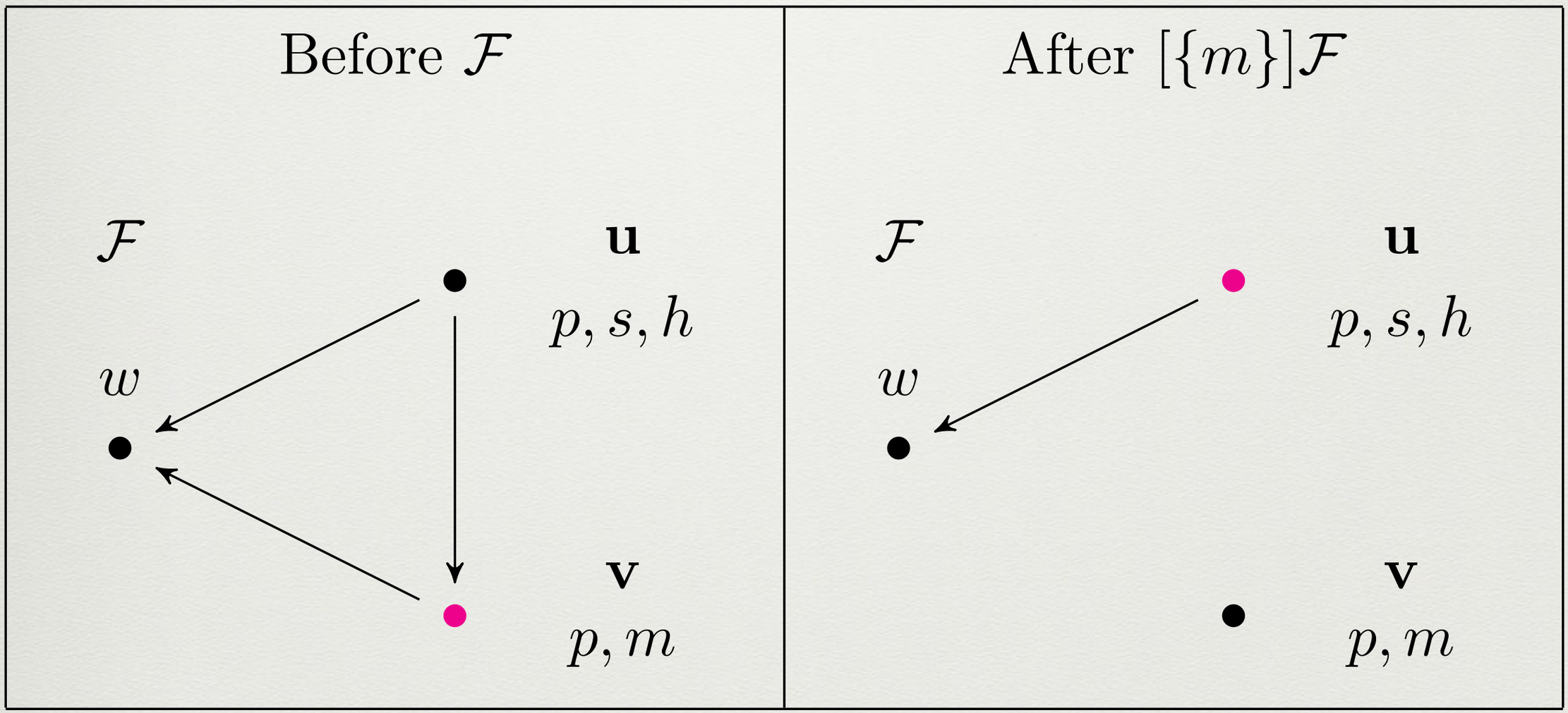
Were I to strike the match,
it would light.

Back to the Fine Model



$$\mathcal{F}, w \models [p, \{m\}]h$$

Ceteris Paribus & Dynamics



Universe of Discourse

p = "Nixon pushes the button,"
s = "the missile successfully launches,"
m = "a miracle prevents the missile being launched,"
h = "a nuclear holocaust occurs,"

- Kit Fine (1975): If Nixon had pushed the button (p), there would have been a nuclear holocaust (h).
- Let p and h vary (UD), keep everything else equal.

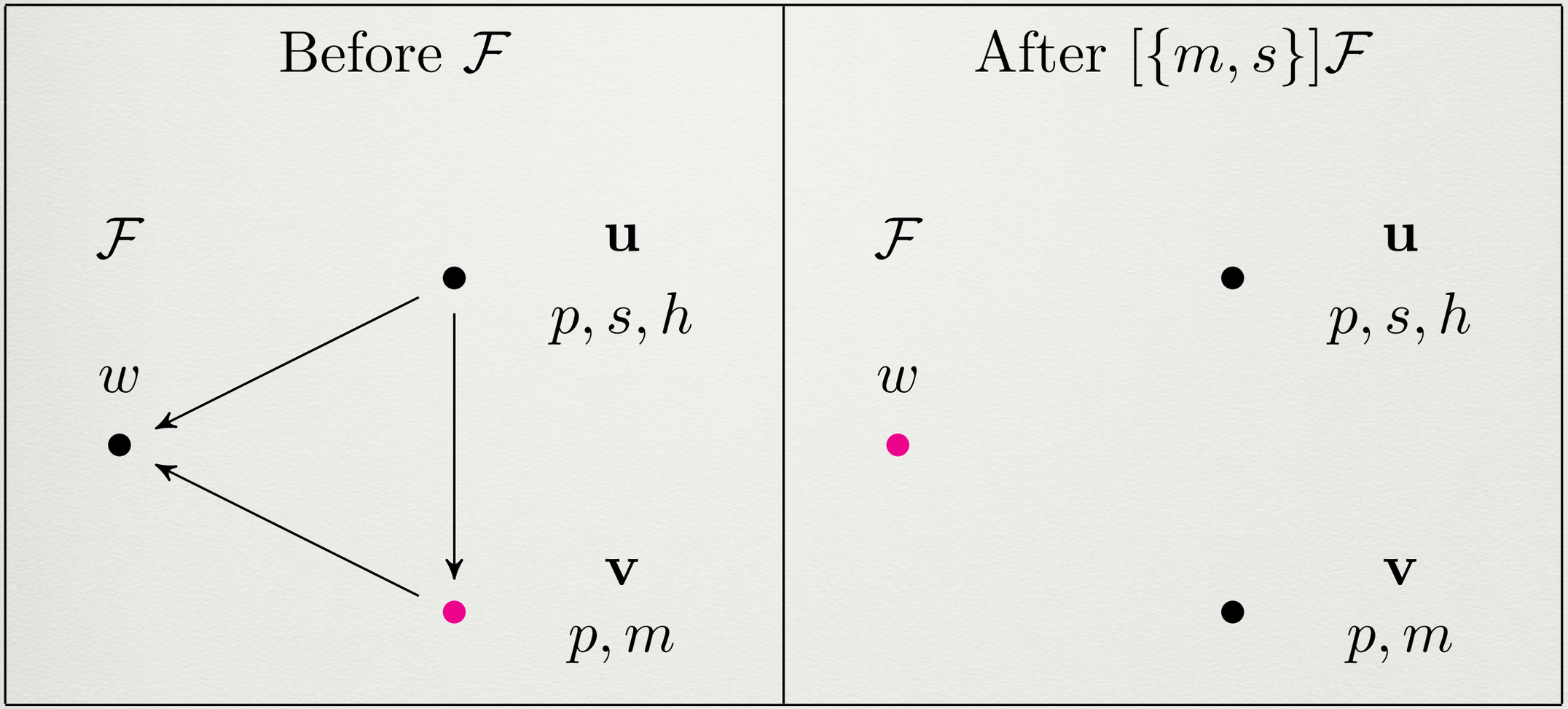
$$[p, \{m, s\}]h$$

Universe of Discourse

$$
\begin{aligned}
\mathsf{UD}(p) &= \{p\} \\
\mathsf{UD}(\neg\varphi) &= \mathsf{UD}(\varphi) \\
\mathsf{UD}(\varphi \vee \psi) &= \mathsf{UD}(\varphi) \cup \mathsf{UD}(\psi) \\
\mathsf{UD}([\varphi, \Gamma]\psi) &= \mathsf{UD}(\varphi) \cup \mathsf{UD}(\Gamma) \cup \mathsf{UD}(\psi) \\
\mathsf{UD}(\{\gamma_1, \ldots, \gamma_n\}) &= \mathsf{UD}(\gamma_1) \cup \cdots \cup \mathsf{UD}(\gamma_n).
\end{aligned}
$$

$$[\varphi, \mathsf{PROP} \setminus (\mathsf{UD}(\varphi) \cup \mathsf{UD}(\psi))]\psi$$

Strict CP



$$\mathcal{F}, w \models [p, \{m, s\}]h$$
$$\mathcal{F}, w \models [p, \{m, s\}]\neg h$$

Relaxing the ceteris paribus clause (Naive counting)

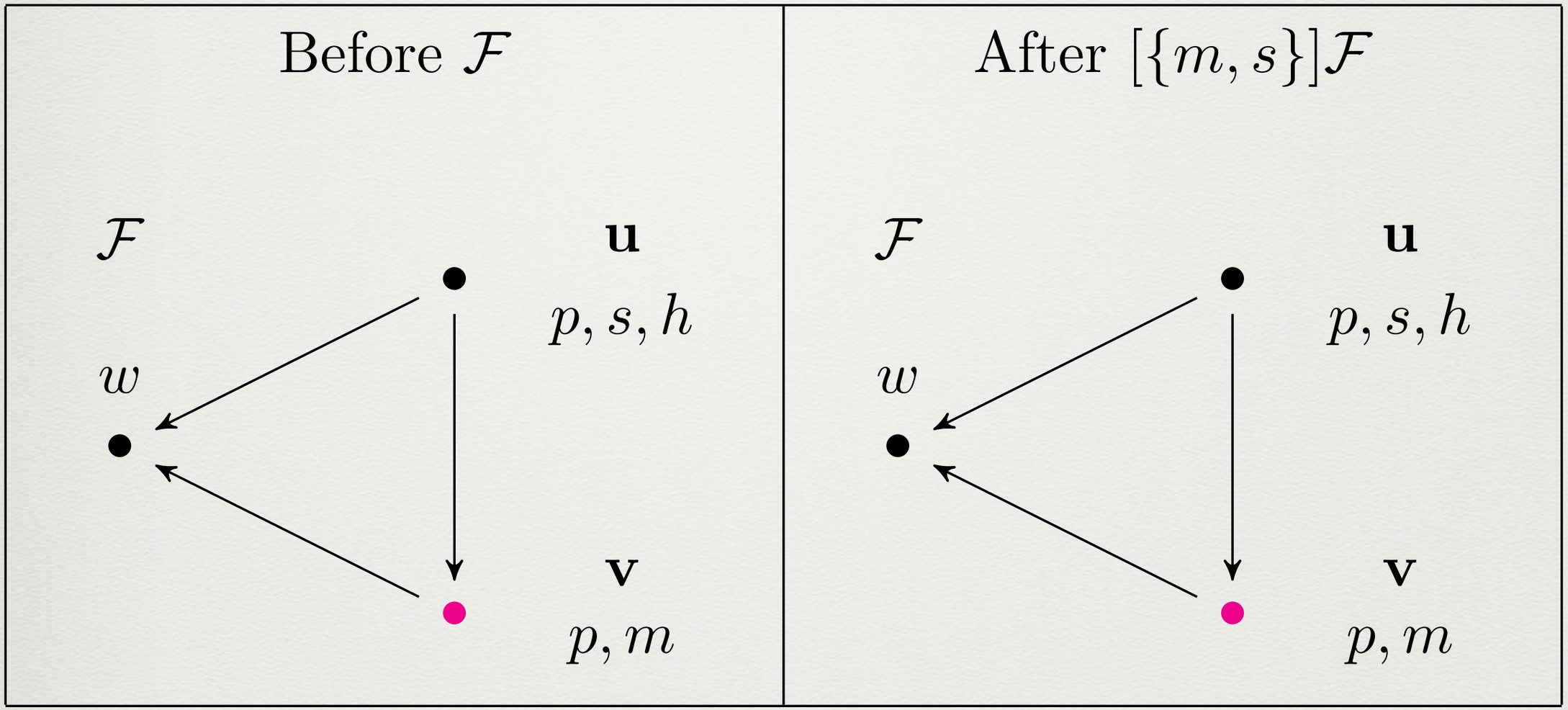
- ψ is true in every most similar φ-world that keeps Γ equal *as much as possible*.

$$A_\Gamma^{\mathcal{M}}(u,v) = \{\gamma \in \Gamma : \mathcal{M}, u \models \gamma \text{ iff } \mathcal{M}, v \models \gamma\}$$

$$u \preceq_w^\Gamma v \text{ iff}$$

$$\text{either } |A_\Gamma^{\mathcal{M}}(u,w)| > |A_\Gamma^{\mathcal{M}}(v,w)|,$$
$$\text{or } |A_\Gamma^{\mathcal{M}}(u,w)| = |A_\Gamma^{\mathcal{M}}(v,w)| \text{ and } u \preceq_w v.$$

Naive Counting (NC)



$$A^{\mathcal{F}}_{\{m,s\}}(w, u) = \{m\}$$
$$A^{\mathcal{F}}_{\{m,s\}}(w, v) = \{s\}$$

$$\mathcal{F}, w \models [p, \{m, s\}]\neg h$$

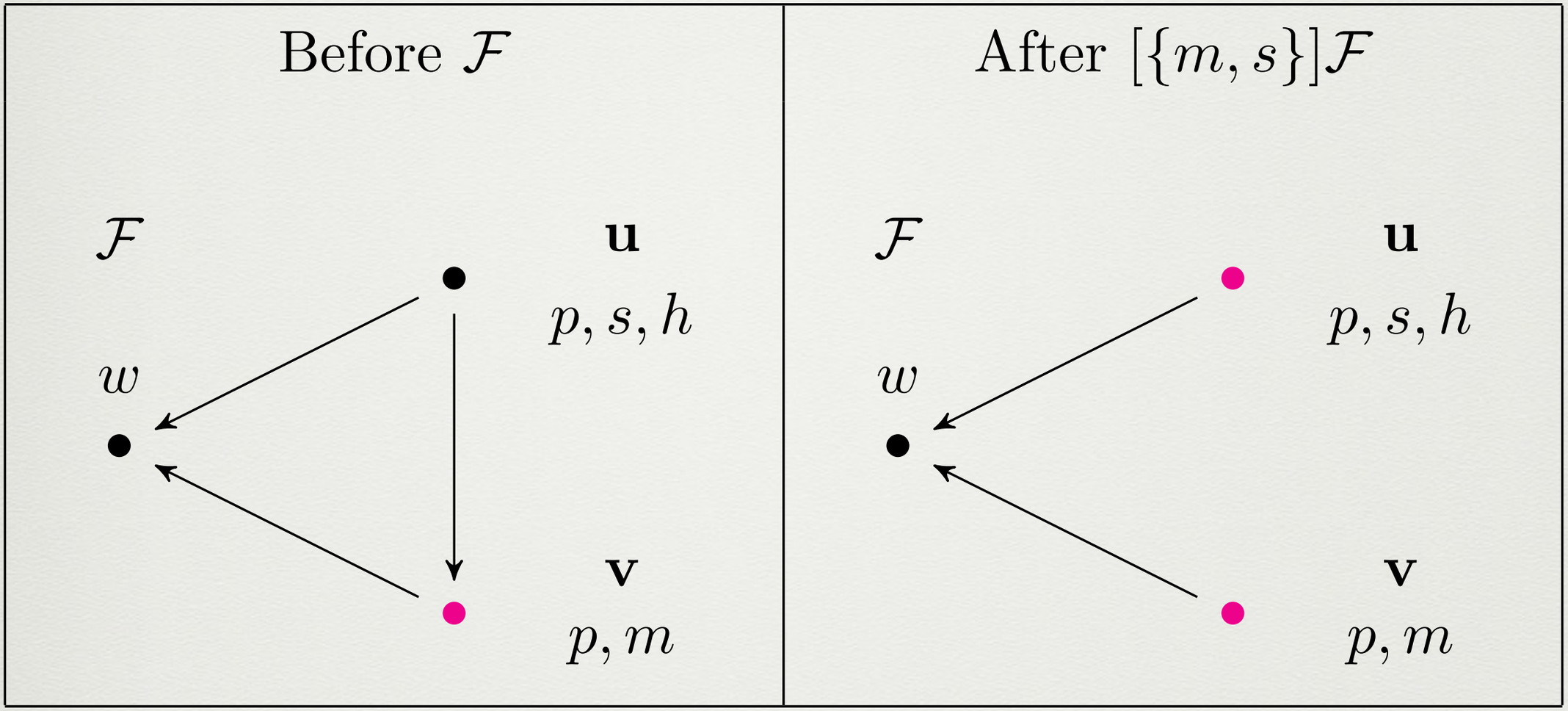
Relaxing the ceteris paribus clause (Maximal supersets)

- ψ is true in every most similar φ-world that keeps Γ equal as much as possible.

$$A_\Gamma^{\mathcal{M}}(u,v) = \{\gamma \in \Gamma : \mathcal{M}, u \models \gamma \text{ iff } \mathcal{M}, v \models \gamma\}$$

$$u \sqsubseteq_w^\Gamma v \text{ iff}$$

$$\text{either } A_\Gamma^{\mathcal{M}}(v,w) \subset A_\Gamma^{\mathcal{M}}(u,w),$$
$$\text{or } A_\Gamma^{\mathcal{M}}(v,w) = A_\Gamma^{\mathcal{M}}(u,w) \text{ and } u \preceq_w v.$$



$$A^{\mathcal{F}}_{\{m,s\}}(w, u) = \{m\}$$
$$A^{\mathcal{F}}_{\{m,s\}}(w, v) = \{s\}$$

$$\mathcal{F}, w \not\models [p, \{m, s\}]h$$

Comparative Possibility

$$\varphi \preceq \psi \quad = \quad \varphi \text{ is at least as } \textit{possible} \text{ as } \psi.$$

i.e., each ψ-world can be matched by a *better* φ-world.

$$\begin{aligned}
\varphi \prec \psi &:= \neg(\psi \preceq \varphi) \\
\Diamond\varphi &:= \varphi \prec \bot \\
\varphi \mathrel{\Box\!\!\rightarrow} \psi &:= \Diamond\varphi \rightarrow (\varphi \wedge \psi) \prec (\varphi \wedge \neg\psi)
\end{aligned}$$

Comparative Similarity, CP

$$
\begin{array}{lcl}
u \equiv_\Gamma v & \text{iff} & \forall \gamma \in \Gamma : M, u \models \gamma \;\; \text{iff} \;\; M, v \models \gamma \\
[w]_\Gamma & = & \{u \in W_w : w \equiv_\Gamma u\} \\
\trianglelefteq_w^\Gamma & := & \preceq_w \cap ([w]_\Gamma \times [w]_\Gamma)
\end{array}
$$

$$
\begin{array}{lcl}
\varphi \trianglelefteq^\Gamma \psi & = & \varphi \;\; \text{is at least as possible as} \;\; \psi, \\
 & & \text{keeping} \;\; \Gamma \;\; \text{equal}
\end{array}
$$

Completeness

- Strict CP:

$$\varphi \trianglelefteq^{\Gamma} \psi \leftrightarrow \bigwedge_{\overline{\Gamma} \in \mathbf{\Gamma}} [\overline{\Gamma} \rightarrow (\varphi \wedge \overline{\Gamma}) \preceq (\psi \wedge \overline{\Gamma})]$$

- Naive Counting

$$\varphi \preceq^{\Gamma} \psi \leftrightarrow \bigwedge_{\overline{\Gamma} \in \mathbf{\Gamma}} (\overline{\Gamma} \rightarrow \bigwedge_{\Lambda \subseteq \overline{\Gamma}} ([(\varphi \wedge \Lambda) \vee (\bigvee_{|\Lambda| < |\Sigma| \leq |\overline{\Gamma}|} \Sigma \wedge \varphi)] \preceq \psi \wedge \Lambda))$$

- Maximal Supersets

$$\varphi \sqsubseteq^{\Gamma} \psi \leftrightarrow \bigwedge_{\overline{\Gamma} \in \mathbf{\Gamma}} (\overline{\Gamma} \rightarrow \bigwedge_{\Lambda \subseteq \overline{\Gamma}} ([(\varphi \wedge \Lambda) \vee (\bigvee_{\Lambda \subset \Sigma \subseteq \overline{\Gamma}} \Sigma \wedge \varphi)] \preceq \psi \wedge \Lambda))$$

Future Work

- Indeterminism. 😕
- Most Counterfactuals are false (Alan Hájek) 😦
- Infinite CP sets. 😖
- Applications in game theory? 🙏🏿