Saussurean Compositionality

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... the semantics works in tandem with the syntax: each syntactic rule which predicts the existence of some well-formed expression (as output) is paired with a semantic rule which gives the meaning of the output expression in terms of the meaning(s) of the input expressions. This is what we mean by Direct Compositionality.

Thus every expression of a language—including the basic expressions (the words)—can be seen as a triple ⟨[sound], syntactic category, meaning⟩. ... A rule is thus something which takes one or more triples as input and yields one as output. (Pauline Jacobson, Compositional Semantics. An Introduction to the Syntax-Semantics Interface)
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Hence this paper.
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*Suppose we think of a language as a collection of form-meaning pairs, where the meanings are concepts in a given conceptual system.* (539)
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Interestingly, Lakoff takes this to support non-compositionality.

– Syntactic categories are not autonomous, nor are they completely predictable form semantic considerations.
– The meanings of whole grammatical constructions are not computable by general rules from the meanings of their parts. (582)
Sign-based grammars, cont

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I will start with a few brief remarks on the second (larger and less precise) question.

Then I will address the first question in some detail, and draw a few conclusions.
Language as a relation between form and content

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So grammar rules (or constraints) generate the well-formed signs, which constitute the language $L$. 
Sign-based ambiguity and synonymy

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$$\pi_1(\langle e, m \rangle) = e \text{ and } \pi_2(\langle e, m \rangle) = m$$

**NB** There are no corresponding terms on the meaning side.
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Ambiguity in the standard account

By the **standard account** we mean syntax in the form of a grammar

\[ E = (E, A, f^E)_{f \in \Sigma} \]

which generates \( E \) from the atoms (lexical items) in \( A \subseteq E \) via the functions (grammar rules) \( f^E \) (where \( \Sigma \) is a **signature**),
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\[ \mu : GT_E \rightarrow M \]

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A sign-based account, on the other hand, simply has the two distinct signs $\langle bank, m_1 \rangle$ and $\langle bank, m_2 \rangle$ in the language.
What about structural ambiguity?

On the standard account, terms (analysis trees) are used for structural ambiguity: e.g. with the rules

\[ N \rightarrow N \quad \text{(rule } f) \]
\[ N \rightarrow A \ N \quad \text{(rule } g) \]

and atoms *old*, *men*, *women*, we get two distinct terms:

\[ t = f(g(\text{old, men}), \text{women}) \]
\[ u = g(\text{old, } f(\text{men, women})) \]

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A compositional semantics \( \mu \) yields

\[ \mu(t) = r_f(r_g(m_o, m_m), m_w) = (m_o \cap m_m) \cup m_w = m \]
\[ \mu(u) = r_g(m_o, r_f(m_m, m_w)) = m_o \cap (m_m \cup m_w) = m' \]

Since there are distinct terms (analysis trees) we can get distinct meanings.
Structural ambiguity, cont.

On the Saussurean account, we have rules $F, G$ corresponding to $f, g$ and $\mu$ for generating signs:

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F(\langle e_1, m_1 \rangle, \langle e_2, m_2 \rangle) = \langle e_1 \text{ and } e_2, m_1 \cup m_2 \rangle \\
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So $\langle \text{old men and women}, m \rangle, \langle \text{old men and women}, m' \rangle$ is a structural ambiguity.

**Conclusion:** In the sign-based format, terms (analysis trees) are not needed to account for lexical or structural ambiguity; it suffices with rules that apply to pairs.
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(These are called ‘non-compositional idioms’ in Nunberg, Sag, and Wasow’s classical 1994 paper; there are various tests by which they can be recognized, e.g. they do not passivize.)
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Consider idioms that have syntactic but no semantic structure; a typical example is *kick the bucket*.

(These are called ‘non-compositional idioms’ in Nunberg, Sag, and Wasow’s classical 1994 paper; there are various tests by which they can be recognized, e.g. they do not passivize.)

Let’s agree that the phrase *kick the bucket* has the following characteristics:
kick the bucket

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W-hl (2002) considered the ‘idiom extension problem’: suppose a phrase acquires an additional idiomatic meaning—how can we extend the grammar and the semantics while preserving compositionality?
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For this kind of ambiguity, one suggestion was to add a **new name for the same rule** (function): say \( f_I \), with \( f^E = f^E_I \).
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For this kind of ambiguity, one suggestion was to add a new name for the same rule (function): say \( f_I \), with \( f^E = f_I^E \).

So the same rule \( (f^E) \) generates the idiomatic *kick the bucket*, the language has the same expressions as before, but different terms, and
\[
\mu(f(\text{kick, the bucket})) \neq \mu(f_I(\text{kick, the bucket}))
\]
(where \( r_{f_I}(m_0, m_1) = \text{DIE} \)).
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Now we have two different rules on pairs:

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Learning the idiom is simply learning the new rule \(f_j^L\)
Two algebraic formats

We saw that the classical set-up can be viewed as a **syntactic algebra**

$$E = (E, A, f^E)_{f \in \Sigma}$$

generating $E$ from the atoms in $A \subseteq E$ via the functions $f^E$, and the **semantics** as a partial function

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Similarly, a sign-based setting consists of a **pair grammar**

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The grammar functions take care of both syntax and semantics (‘in tandem’).
Compositionality in the classical setting

Given \( E \), a semantics \( \mu \) for \( E \) is \textit{compositional} if

\[
\text{Funct}(\mu) \quad \text{for each } f \in \Sigma \text{ there is an operation } r_f \text{ such that if } f(t_1, \ldots, t_n) \in \text{dom}(\mu), \text{ then } \mu(f(t_1, \ldots, t_n)) = r_f(\mu(t_1), \ldots, \mu(t_n))
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Equivalently, letting $t \equiv_\mu u$ iff $\mu(t), \mu(u)$ are both defined and $\mu(t) = \mu(u)$

we have (provided subterms of meaningful terms are always meaningful) the substitution version:

$\text{Subst}(\equiv_\mu)$ If $s[t_1, \ldots, t_k]$ and $s[u_1, \ldots, u_k]$ are both in $\text{dom}(\mu)$, and $t_i \equiv_\mu u_i$ for $i = 1, \ldots, n$, then $s[t_1, \ldots, t_k] \equiv_\mu s[u_1, \ldots, u_k]$
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(Here $t_1, \ldots, t_k$ are **disjoint subterm occurrences** in $s$.)
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(ii) (Saussurean) The meaning of a complex sign is determined by the meanings of its immediate constituent signs and the mode of composition.

(ii) makes good sense: each sign has a unique meaning, constituency for signs is defined via the term algebra for $L = (L, A_L, f^L)_{f \in \Delta}$, and the modes of composition are the grammar rules $f^L$. 
Three kinds of Saussurean compositionality

In a sign-based framework, there are essentially three notions of compositionality:

(i) The meaning of a sign is determined by the meanings of the constituent signs.

(ii) The expression of a sign is determined by the expressions of the constituent signs. Kracht calls this autonomy.

(iii) Both (i) and (ii) hold. Kracht calls this independence.
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No. This is built into the sign-based format, where complex signs—and hence their meanings—are determined via the grammar rules by (the expressions and meanings of) their immediate constituent signs.
A contrast?

Does the last point mark a contrast between the standard format and the sign format?
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Does the last point mark a contrast between the standard format and the sign format? In the former you can have recursive (not necessarily compositional!) composition operators:

\[ \text{Rec(} \mu \text{)} \quad \text{for each } f \in \Sigma \text{ there is an operation } s_f \text{ such that if} \]
\[ f(t_1, \ldots, t_n) \in \text{dom}(\mu), \text{ then} \]
\[ \mu(f(t_1, \ldots, t_n)) = s_f(t_1, \ldots, t_n, \mu(t_1), \ldots, \mu(t_n)) \]
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\text{Rec}(\mu) only has bite if the \( s_f \) are required to be computable in some suitable sense (computation on meanings).

In other words: once you have a sign-based grammar, or a standard grammar + semantics, it is trivial in both cases that the meanings of complex expressions are determined (not computable!) by the meanings of their immediate parts and the parts themselves (and the mode of composition).
Saussurean compositionality: precise versions

Recall that we use partial grammar functions, since we avoid categories.
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This entails that there are variants of the precise formulation of the three Saussurean notions, depending on when the composition operations are taken to be defined. Here we choose the following, for \( L = (L, A_L, f^L)_{f \in \Delta} \):

- \( L \) is **compositional** iff for each \( f \in \Delta \) there is an operation \( r_{2f} \) such that for \( \langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle \in L \), if \( f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle) \) is defined, then \( \pi_2(f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle)) = r_{2f}(m_1, \ldots, m_n) \); undefined otherwise.

- \( L \) is **autonomous** iff for each \( f \in \Delta \) there is an operation \( r_{1f} \) such that for \( \langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle \in L \), if \( f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle) \) is defined, then \( \pi_1(f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle)) = r_{1f}(e_1, \ldots, e_n) \); undefined otherwise.

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otherwise $r_{1f}(e_1, \ldots, e_n)$ or $r_{2f}(m_1, \ldots, m_n)$ is undefined.
A substitution version of Saussurean compositionality

**Definition**

L is **right-centered** if for all $f \in \Delta$, whenever $f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle)$ is defined and $\langle e'_1, m_1 \rangle, \ldots, \langle e'_n, m_n \rangle \in L$, $f^L(\langle e'_1, m_1 \rangle, \ldots, \langle e'_n, m_n \rangle)$ is also defined.
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and consider the following property:

\( \text{Subst}(\equiv^{2,L}) \) If \( p[q_1, \ldots, q_n] \) and \( p[q'_1, \ldots, q'_n] \) are both in \( GT_L \), and \( q_i \equiv^{2,L} q'_i \) for \( 1 \leq i \leq n \), then \( p[q_1, \ldots, q_n] \equiv^{2,L} p[q'_1, \ldots, q'_n] \).
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**Fact**

L is compositional iff it is right-centered and Subst($\equiv^{2,L}$) holds (similarly for autonomy and independence).
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Recall also our earlier notions of a synonymy pair $(\langle e, m \rangle, \langle e', m \rangle)$, and an ambiguity pair $(\langle e, m \rangle, \langle e, m' \rangle)$.
Compositionality in the two frameworks

Application: trivial Saussurean compositionality

Recall that in the classical format, if the meaning function $\mu$ is one-one, i.e. if there are no non-trivial synonymies, then $\mu$ is trivially compositional (use the substitution version).

(Which again emphasizes the need for computability.)

Recall also our earlier notions of a synonymy pair ($\langle e, m \rangle, \langle e', m \rangle$), and an ambiguity pair ($\langle e, m \rangle, \langle e, m' \rangle$).

Fact

If $L$ has no non-trivial synonymies (ambiguities), then any pair grammar for $L$ is compositional (autonomous).
Notational variants?

Let $L = (L, A_L, f^L)_{f \in \Delta}$ be a pair grammar, where $L \subseteq E \times M$. 

The two main problems are:

- the different treatments of ambiguity;
- translation works only when compositionality is assumed.
Comparing the two frameworks

Notational variants?

Let \( L = (L, A_L, f^L)_{f \in \Delta} \) be a pair grammar, where \( L \subseteq E \times M \). If there was a canonical way to associate a classical grammar \( E_L = (E, A, f^E)_{f \in \Delta} \) and a semantics \( \mu_L \) for \( E_L \) with values in \( M \) such that

(a) \( L \) is compositional iff \( \mu_L \) is compositional;
(b) \( \langle e, m \rangle \in L \) iff there is \( t \in GT_{E_L} \) such that \( val(t) = e \) and \( \mu_L(t) = m \),

then we might say that the classical version is a notational variant of the sign-based one.
Comparing the two frameworks

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And vice versa, if starting from a grammar $E = (E, A, f^E)_{f \in \Sigma}$ and a semantics $\mu$ for $E$ with values in $M$, we could always find a pair grammar $L_{E, \mu}$ such that conditions corresponding to (a) and (b) hold.
Comparing the two frameworks

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Comparing the two frameworks
Translations I

From pair grammars to classical grammars + semantics 1

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Comparing the two frameworks
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Lemma

$E_L = (E, AE_L, r_{1f})_{f \in \Delta}$ is a syntactic algebra: $E$ is generated from $AE_L$ via the $r_{1f}$. 
Comparing the two frameworks

Translations I

From pair grammars to classical grammars $+$ semantics I

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Proof.

By independence (essentially autonomy) of $L$. $\square$
Comparing the two frameworks
Translations I

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Let \( L = (L, A_L, f^L)_{f \in \Delta} \) be an independent pair grammar, so that for each \( f \in \Delta \), the composition operations \( r_{1f} \) and \( r_{2f} \) are given. Let

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But to obtain a semantics, we must assume that there is no lexical ambiguity, in the following sense:

\((NLA)\) \( \langle e, m \rangle \in A_L \) implies \( \forall m' (\langle e, m' \rangle \in L \Rightarrow m' = m) \)
Comparing the two frameworks  
Translations I

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Comparing the two frameworks Translations I

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Thus, \( a \in AE_L \) has a unique meaning \( \mu_L(a) \in M \). When forming \( GT_{E_L} \) we can identify the atomic terms with the atomic expressions, and define the complex (grammatical) terms in \( GT_{E_L} \) as usual via the \( r_{1f} \), simultaneously with the surjective homomorphism, call it \( val^E \), from terms to \( E \).
Finally, we inductively extend $\mu_L$ to (some) terms $t$ in $GT_{E_L}$, s.t. that for each $t$ in $\text{dom}(\mu_L)$, $\langle \text{val}^E(t), \mu_L(t) \rangle \in L$:
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Suppose $t = f(t_1, \ldots, t_n)$, where (ind. hyp.) $\text{val}^E(t_i) = e_i$, $\mu_L(t_i) = m_i$, and $\langle e_i, m_i \rangle \in L$. Since $t$ is well-formed, $r_1f(e_1, \ldots, e_n)$ is defined. If $r_2f(m_1, \ldots, m_n)$ is also defined, then, by independence, so is $f^L(\langle e_1, m_1 \rangle, \ldots, \langle e_n, m_n \rangle)$, say with value $\langle e, m \rangle$, and we let $\mu_L(t) = m = r_2f(m_1, \ldots, m_n)$. If not, $\mu_L(t)$ is undefined.
Comparing the two frameworks

Translations I

From pair grammars to classical grammars + semantics

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Theorem

If $L = (L, A_L, f^L)_{f \in \Delta}$ is independent and has no lexical ambiguity, then $\mu_L$ is a compositional semantics for $E_L$, and the function

$$h(t) = \langle \text{val}^E(t), \mu_L(t) \rangle$$

is a surjective homomorphism from $GT_{E_L}$ to $L$. 
Finally, we inductively extend $\mu_L$ to (some) terms $t$ in $GT_{E_L}$, s.t. that for each $t$ in $\text{dom}(\mu_L)$, $\langle \text{val}^E(t), \mu_L(t) \rangle \in L$:

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**Proof.**

For $f \in \Delta$, the composition operation $r_f$ is $r_2f$. 

\(\square\)
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Essentially, the result says that if we start with a sign-based grammar and want to turn it into the classical format, we shall only succeed if the sign-based grammar is independent.
Comparing the two frameworks

Translations I

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Essentially, the result says that if we start with a sign-based grammar and want to turn it into the classical format, we shall only succeed if the sign-based grammar is independent.

Also, we must start by introducing new expressions ($bank_1$, $bank_2$, ...) to eliminate lexical ambiguity.
Now let a grammar $E = (E, A, f^E)_{f \in \Sigma}$ be given, with its corresponding set $GT_E$ of grammatical terms and surjective homomorphism $val: GT_E \rightarrow E$, and a **compositional** semantics $\mu$ for $E$ with values in $M$. 
Now let a grammar $\mathbf{E} = (E, A, f^E)_{f \in \Sigma}$ be given, with its corresponding set $GT_E$ of grammatical terms and surjective homomorphism $val: GT_E \to E$, and a compositional semantics $\mu$ for $\mathbf{E}$ with values in $M$.

New atomic terms may have been added.
Comparing the two frameworks

From classical grammars + semantics to pair grammars

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New atomic terms may have been added. We assume the set $\overline{A}$ of atomic terms is well-behaved in the following sense:

(i) $t \in \overline{A}$ iff $val(t) \in A$

(ii) if $t, t' \in \overline{A}$, $t \neq t'$, and $val(t) = val(t')$, then $\mu(t) \neq \mu(t')$

(iii) $\overline{A} \subseteq dom(\mu)$
Comparing the two frameworks
Translations II

From classical grammars $+$ semantics to pair grammars

Now let a grammar $\mathbf{E} = (E, A, f^\mathbf{E})_{f \in \Sigma}$ be given, with its corresponding set $GT_\mathbf{E}$ of grammatical terms and surjective homomorphism $\text{val}: GT_\mathbf{E} \to E$, and a compositional semantics $\mu$ for $\mathbf{E}$ with values in $M$.

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Then define:

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Comparing the two frameworks

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Then define:

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L_{E,\mu} = \{ \langle \text{val}(t), \mu(t) \rangle : t \in \text{dom}(\mu) \} \\
At_{E,\mu} = \{ \langle e, m \rangle \in L_{E,\mu} : e \in A \}
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Comparing the two frameworks

From classical grammars + semantics to pair grammars 1

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and:
From classical grammars + semantics to pair grammars 2

For $f \in \Sigma$, define a partial function $f^{E,\mu}$ from $(L_{E,\mu})^n$ to $L_{E,\mu}$: given

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if $f(t_1, \ldots, t_n)$ is defined and in $\text{dom}(\mu)$; undefined otherwise.
Comparing the two frameworks  
Translations II

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    \langle \text{val}(f(t_1, \ldots, t_n)), \mu(f(t_1, \ldots, t_n)) \rangle
\end{align*}
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if \( f(t_1, \ldots, t_n) \) is defined and in \( \text{dom}(\mu) \); undefined otherwise.

This is well-defined precisely because \( \mu \) is compositional.
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L_{E,\mu} = (L_{E,\mu}, A_{t_{E,\mu}}, f^{E,\mu})_{f \in \Sigma}.
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**Theorem**

(a) If \( \mu \) is a compositional semantics for \( E \) and \( GT_E \) has well-behaved atomic terms, then \( L_{E,\mu} \) is independent (with operations \( r_1f = f^E \) and \( r_2f = r_f \)), and we have \( \langle e, m \rangle \in L_{E,\mu} \) iff for some \( t \in GT_E \), \( \text{val}(t) = e \) and \( \mu(t) = m \).
From classical grammars + semantics to pair grammars 2

For \( f \in \Sigma \), define a partial function \( f^{E, \mu} \) from \((L_{E, \mu})^n\) to \( L_{E, \mu} \): given \( \langle val(t_1), \mu(t_1) \rangle, \ldots, \langle val(t_n), \mu(t_n) \rangle \) in \( L_{E, \mu} \), let

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(b) If in addition \((E, \mu)\) has no lexical ambiguity (so \( \overline{A} = A \)), then, applying the previous construction to \( L_{E, \mu} \), we get back what we started from, i.e. \( E_{L_{E, \mu}} = E \) and \( \mu_{L_{E, \mu}} = \mu \).
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Then you can give a corresponding pair grammar, which will be independent provided your semantics was compositional.
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We could conclude that, modulo ambiguity issues, the independent Saussurean pair grammar is indeed a notational variant (but in a rather weaker sense than I used before) of a classical style grammar and a compositional semantics.
Classical grammars + semantics vs. pair grammars

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First, it gives an actual pair grammar $L$ (for various interesting features of quotation).

More exactly, $L$ generates triples of the form $⟨e, X, m⟩$ (where $X$ is a syntactic category) with the quotation rule as a unary function on triples $q_L(⟨e, X, m⟩) = ⟨e, X, ⟨e, X, m⟩⟩$.

Second, $L$ is claimed to be compositional. This is taken as an immediate corollary of the Direct Compositionality idea of syntax and semantics working ‘in tandem’.

Third, however, it is clear that the grammar $L$, although autonomous, is not compositional in our sense: precisely because of the rule $q_L$, substituting (terms denoting) triples with the same meaning does not preserve meaning. This illustrates the unclarity of the ‘in tandem’ idea: if it just means the pair (or triple) format, it says nothing about compositionality.
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Similarly, one can at least imagine letting the expression of a pair (triple) depend also on the meanings of its parts, in certain special cases (I don’t have any convincing examples of this, but cf. Lakoff).
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Although the existence of suitable composition functions can now be trivial, their computability is not.
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So far the only case of a realistic and fully specified computable but non-compositional grammar I have seen in the literature is the example by Potts just mentioned.
Further directions

NB The standard arguments for compositional semantics, in terms of learnability, productivity, systematicity, etc., show (at most) that computable (recursive) semantics is required.
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TO DO: look at
- complexity results for general compositionality;
- evaluate the pair format vs. the standard format from this perspective.
THANK YOU